# Discussion of “Collateral Consequences: The Debt Channel of Investment Tax Incentives”

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# The paper uncovers a striking balance-sheet response

DRRP policy: Portugal 2014–22. Claims analyzed: 2014–17; outcomes through 2019. SMEs reserve profits, receive a 10% tax credit, and invest the reserve in new tangible assets within two years.

| +2.2 to 4.7 pp | +20 to 44% | +5 to 30% |
|---|---|---|
| tangible investment rate | bank debt; secured moves similarly | employment; wages up to +10% |

The tension: debt rises, while liabilities relative to assets fall by 2–10 pp.

**This discussion:** a dynamic, structural model of the firm's choices, used to look critically at each of these three numbers.

# Point 1: the credit is tiny; payouts must do the work user cost derivation

The model has a price channel and a wealth channel; for a collateral-constrained firm, this slide quantifies the wealth channel:

$$\Delta I^W = \underbrace{(cs+q)}_{\substack{\text{new money in the firm:} \\ \text{credit + payouts kept in}}} \times \underbrace{\frac{1}{1-\theta}}_{\substack{\text{financing multiplier} \\ \approx 2.5}}$$

$I^W$: investment over the two-year statutory window; $q \equiv -\Delta d \in [0, s]$: the drop in the dividend $d$ relative to the no-claim path (not Tobin's $q$); each euro also brings $\theta/(1-\theta) \approx 1.5$ of new debt.

| Payout counterfactual | new money ($cs+q$) | investment ($\Delta I^W/A$) | new debt ($\Delta B$) |
|---|---|---|---|
| Retained anyway: $q=0$ | €1.5k | 0.7 pp | €2.3k |
| Full displacement: $q=s$ | €16.8k | 7.4 pp (3.7/yr) | €25k |

Illustration: $s \approx$ €15.3k, $c=0.1$, $\theta=0.6$, $A \approx$ €570k. The price-margin case (10% credit as a subsidy while the investment

# Point 2: the collateral evidence points to cash, not machines evidence

In the model every euro of debt is collateralized, so secured and total debt are one object:

$$\underbrace{b' = \theta k'}_{\text{all borrowing is against assets}} \quad \Longrightarrow \quad \Delta B^{\text{sec}} \;=\; \Delta B \;=\; \underbrace{\frac{\theta}{1-\theta}\,(cs+q)}_{\text{Point 1's new debt}}$$

Data: total debt $+20$ to $44\%$, secured $+22$ to $41\%$, secured share flat at $87\%$ (€264k of €302k). With 87% already secured, similar proportional responses are not separately informative.

- **What moved is the wrong collateral.** Pledging probability $+9$ pp, all of it financial (cash, deposits, securities); real assets flat. Consistent with pledging liquidity, Point 1's money; it is not evidence of new machines as collateral.
- **And it moves too early.** The 2014 cohort's debt rises in the claim year, before eligible investment exists.

What would settle it: link loan originations to collateral type and asset purchases.

# Point 3: employment follows capital; wages are about hiring labor

The labor block ties employment to capital with no free parameters:

$$\Delta \log \ell \;=\; \underbrace{\frac{\alpha}{1-\gamma}}_{\substack{\text{scale response} \\ \approx 0.625}} \Delta \log k \;\approx\; \underbrace{13\%}_{\text{for } \Delta \log k \approx 0.20}$$

Data: employment $+5$ to $30\%$ (the model's $13\%$ sits inside); average real wages up to $+10\%$.

- **Employment is not a separate finding.** Scale adjustment alone delivers it; capital and employment co-moving says nothing about the elasticity of substitution.
- **The wage number is about who is hired.** The benchmark takes the wage as given; a $10\%$ raise for incumbents costs $\approx$ €12k per year against a financing surplus of $\approx$ €3.4k. Composition, most plausibly.

What would settle it: employee records; split incumbent wage changes from new hires.

# The model is a measurement plan model

Each constraint in the firm's problem has a multiplier, and each multiplier has an observable counterpart:

| Constraint (multiplier) | Observable | In the data |
|---|---|---|
| Payout lock ($\rho$) | dividends around the claim | is $q > 0$? Point 1's missing number. |
| Investment requirement ($\xi$) | window investment vs. $s$ | bunching at $I^W = s$ indicates a binding requirement |
| Tax cap ($\zeta$) | reserves vs. $\bar{s}(T)$ | bunching on the max-credit line (paper's Fig. 4) |

All of these are firm choices: estimate them off predetermined exposure (2010–13 profits and eligibility), not claiming or claim timing.

Bottom line: this setting can separate the wealth and price channels, once selection and timing are explicit.

# Backup

# Backup: the minimal DRRP-augmented firm problem return

Statutory objects: claim $s \in [0, \bar{s}(T)]$ with $\bar{s}(T) = \min\{€5\text{m},\ 0.25\,T/c\}$; credit $cs$ with $c = 0.1$; a two-year obligation to invest the reserved amount in eligible tangible assets, tracked by an obligation state $a$ with law of motion $a' = \mathcal{G}(a, s, i^e)$, whose feasible set enforces the two-year deadline and prevents double counting of eligible investment $i^e$.

$$V(k, b, a) = \max_{d,\ell,k',b',s,i^e} \left\{ d + \beta V\big(k', b', \mathcal{G}(a, s, i^e)\big) \right\}$$

subject to

$$d + i = (1-\tau)[F(k,\ell) - w\ell] + \tau\delta k - R_\tau b + b' + cs,$$
$$i = k' - (1-\delta)k, \qquad b' \le \theta k',$$
$$d + s \le \Pi^d, \qquad 0 \le s \le \bar{s}(T), \qquad 0 \le i^e \le i.$$

Let $\Omega' \ge 0$ be the marginal continuation cost of adding one euro to the statutory obligation schedule. For a positive claim,

$$\lambda c = \rho + \zeta + \beta\Omega'.$$

The payout lock $d + s \le \Pi^d$ reflects that the reserve is constituted out of distributable profits; $\rho$, $\zeta$ are its multipliers. The reserve is an equity account, not segregated cash.

# Backup: compressed KKT behind the user cost return

Compress the obligation state into the requirement $i \geq s$ (multiplier $\xi$) and write $\Pi(k, \ell) = (1 - \tau)[F(k, \ell) - w\ell] + \tau\delta k$:

$$\begin{aligned}\mathcal{L} = d + \beta V' + \lambda\big[\Pi(k, \ell) - R_\tau b + b' + cs - d - i\big] \\ + \mu(\theta k' - b') + \xi(i - s) + \rho(\Pi^d - d - s) + \zeta(\bar{s} - s),\end{aligned}$$

with $i = k' - (1 - \delta)k$. For interior $d > 0$ and a positive claim $s > 0$:

$$\begin{aligned} d: &\quad 1 = \lambda + \rho, \\ s: &\quad \lambda c = \xi + \rho + \zeta, \\ k': &\quad \lambda - \xi - \theta\mu = \beta V'_k, \\ b': &\quad \lambda - \mu = -\beta V'_b. \end{aligned}$$

Complementary slackness includes

$$\mu(\theta k' - b') = \xi(i - s) = \rho(\Pi^d - d - s) = \zeta(\bar{s} - s) = 0.$$

# Backup: deriving the user cost KKT return

Holding the tax-cap and distributable-profit limits locally fixed in the envelope,

$$V_b = -\lambda R_\tau, \qquad V_k = \lambda\Big[(1-\tau)F_k + 1 - (1-\tau)\delta\Big].$$

Substituting into the $b'$ and $k'$ conditions and writing $m = \mu/\lambda$, $x = \xi/\lambda$:

$$\lambda_t(1 - m_t) = \beta R_\tau \lambda_{t+1},$$
$$\lambda_t(1 - x_t - \theta m_t) = \beta\lambda_{t+1}\{(1-\tau)F_{k,t+1} + 1 - (1-\tau)\delta\}.$$

Dividing the second condition by the first and rearranging with $R_\tau = 1 + (1-\tau)r$,

$$F_{k,t+1} = r + \delta + \frac{R_\tau}{(1-\tau)(1-m_t)}\Big[m_t(1-\theta) - x_t\Big].$$

$x_t = 0$ removes the direct DRRP price wedge; $m_t = x_t = 0$ recovers the standard Hall–Jorgenson benchmark $F_k = r + \delta$.

- Tax cap binds, requirement slack: $x = 0$; no price subsidy, only the net-worth margin ($cs + q$ can lower $m$).
- Requirement binds, cap slack: $x = c - \rho/\lambda \in [0, c]$; the gross subsidy, reduced when the reserve forces payouts down.

# Backup: deriving the investment-rate equation return

With the collateral constraint binding today and tomorrow, $b = \theta k$ and $b' = \theta k'$. The flow budget is

$$d + i = (1-\tau)Y + \tau\delta k - R_\tau b + b' + cs, \qquad k' = (1-\delta)k + i,$$

where $Y = F(k,\ell) - w\ell$ is the pre-tax operating surplus. Substituting the two constraints and solving for $i$:

$$\frac{i}{k} = \frac{(1-\tau)Y/k + 1 - (1-\tau)\delta - \theta R_\tau}{1-\theta} - (1-\delta) + \frac{cs-d}{(1-\theta)k}.$$

Holding the state fixed, a claim $s$ that lowers the dividend $d$ by $q = -\Delta d \geq 0$ relative to the no-claim path changes investment and borrowing by

$$\Delta i = \frac{cs+q}{1-\theta}, \qquad \Delta B = \frac{\theta(cs+q)}{1-\theta}.$$

Define $I^W \equiv \sum_{t \in \text{window}} i_t$, cumulative investment over the two-year statutory window. A one-time injection raises window investment one-for-one with $\Delta i$, so, holding start-of-window assets $A$ fixed,

$$\Delta\left(\frac{I^W}{A}\right) = \frac{cs+q}{(1-\theta)A}.$$

# Backup: labor demand and the wage calculation return

Cobb–Douglas benchmark: $y = zk^{\alpha}\ell^{\gamma}$, $\alpha + \gamma < 1$. Static labor demand:

$$F_{\ell} = w \quad \Longrightarrow \quad \gamma z k^{\alpha} \ell^{\gamma - 1} = w, \qquad \Delta \log \ell = \frac{\alpha}{1 - \gamma} \Delta \log k.$$

- With $\alpha = 0.25$, $\gamma = 0.6$ and $\Delta \log k \approx 0.20$: $\Delta \log \ell \approx 0.125$, inside the paper's 5–30% range. Capital and employment co-moving is not evidence on the elasticity of substitution.
- A 10% increase in incumbent wages would cost about €12k per year at the median payroll (8 workers at €15k). That is large relative to the illustrative financing surplus of about €3.4k ($0.08\times$ €42k): pure incumbent rent-sharing is difficult to reconcile with the calibration.
- Employee records would separate incumbent wage changes from hiring composition; relative $k/\ell$ responses can discipline $\sigma$ only with credible variation in relative factor prices.

Diagnostic magnitudes, not formal bounds. Implied $K_0 \approx$ €210k, i.e. 37% of median assets.

# Backup: what the collateral evidence establishes return

- Secured share among treated observations: $\frac{263.62}{301.86} = 0.873$ (Table B.1 means, 2010–19).
- Probability of pledging collateral rises by up to 9 pp, driven by **financial** collateral; real mortgage and real non-mortgaged categories are approximately unchanged.
- Collateral *value* estimates are small and statistically insignificant overall; real not-mortgaged values are positive but noisy.
- Debt for the 2014 cohort is already higher in the claim year itself (Figure 10a), before the statutory investment window opens.

The pattern is consistent with balance-sheet strengthening and bank screening on liquidity, not yet with new machinery pledged as collateral.