Aggregate Risk in Heterogenous Agent Economies

ECO2101 Lecture 6 | March 18, 2026

Sebastian Dyrda, University of Toronto

Why Aggregate Risk in Heterogeneous-Agent Models?

Aggregate shocks are central to macro:

- Monetary, fiscal, technological, financial shocks drive business cycles
- Standard models study their effects on aggregates (output, consumption, investment)

But agents are heterogeneous:

- Income, wealth, constraints differ across households and firms
- Exposure to shocks is uneven (unemployment, leverage, productivity)

Two-way interaction:

- **Macro** $\rightarrow$ **Micro:** aggregate shocks redistribute income, wealth, and risk
- **Micro** $\rightarrow$ **Macro:** distribution shapes aggregate responses (consumption, investment)

Key question:

Do we need the full distribution to understand aggregate dynamics?

Krusell and Smith (1998): Insight and Limits

Methodological contribution:

- Solve heterogeneous-agent models with aggregate risk
- Key difficulty: the distribution is a state variable
- Solution: approximate the distribution with a small set of moments

Striking result:

Aggregate dynamics are well approximated by mean capital alone

Interpretation:

- Micro heterogeneity has (almost) no effect on macro dynamics *in this environment*

But: limited scope

- Relies on weak interaction between aggregate shocks and the distribution
- In many environments, higher moments matter or aggregation fails
- We cannot, in general, study macro without tracking micro heterogeneity

Aiyagari model with aggregate productivity shocks

- Aggregate and Idiosyncratic Risk. The aggregate productivity shock z only takes two values, $z \in Z \equiv \{z_g, z_b\}$ with $z_b < z_g$.

- We also assume only two values for the individual productivity, $\varepsilon \in E \equiv \{\varepsilon_b, \varepsilon_g\}$ with $\varepsilon_b < \varepsilon_g$. KS have $\varepsilon_b = 0$ (unemployed) and $\varepsilon_g = 1$ (employed).

- Let
$$\pi\left(z', \varepsilon' | z, \varepsilon\right) = \Pr\left(z_{t+1} = z', \varepsilon_{t+1} = \varepsilon' | z_t = z, \varepsilon_t = \varepsilon\right)$$
be the Markov chain that describes the joint evolution of the exogenous shocks.

- Notation allows transition probabilities for ε to depend on z, z'.

Block Representation

The same transition matrix can be written in 2×2 blocks as

$$P = \begin{pmatrix} P_{bb} & P_{bg} \\ P_{gb} & P_{gg} \end{pmatrix}, \qquad P_{zz'} = \begin{pmatrix} \pi(z', \varepsilon_b \mid z, \varepsilon_b) & \pi(z', \varepsilon_g \mid z, \varepsilon_b) \\ \pi(z', \varepsilon_b \mid z, \varepsilon_g) & \pi(z', \varepsilon_g \mid z, \varepsilon_g) \end{pmatrix}.$$

- Rows: current aggregate state $z \in \{z_b, z_g\}$.
- Columns: next aggregate state $z' \in \{z_b, z_g\}$.
- Within each block, entries describe transitions in idiosyncratic risk ε.
- This notation makes transparent how recessions vs booms can change job-finding and separation rates.

Aiyagari model with aggregate productivity shocks

- Production side. From the firm's optimization problem we have

$$\begin{aligned} w &= zF_H(K,H) \\ R &= 1 + zF_K(K,H) - \delta \end{aligned}$$

- Note: prices depend on the K/H ratio, not just on K, but the dynamics of H can be perfectly forecasted, conditional on (z, z'), through π because labor supply is exogenous.
- z is a sufficient statstic for $H \in \{H_b, H_g\}$ (KS, 1998)
- H is time varying, but we know how to forecast it, e.g.:

$$H(z) = \sum_{\varepsilon \in E} \varepsilon \Pi_z(\varepsilon)$$

Aiyagari model with aggregate productivity shocks

- State variables - the two individual states are $(a, \varepsilon) \in S$ and the two aggregate states are $(z, \lambda) \in Z \times \Lambda$ where $\lambda(\cdot)$ is the measure of households across states.
- λ represents the beginning-of-period distribution of wealth and employment status (ε), after this period employment status is realized (wealth is inherited from last period saving decision).
- The individual states are directly budget relevant.
- The aggregate states are needed to compute today's prices.
- The law of motion Ψ is needed to forecast prices.

Aiyagari model with aggregate productivity shocks

- Household Problem in recursive form:

$$
\begin{aligned}
v(a,\varepsilon;z,\lambda) &= \max_{c,a'}\left\{u(c)+\beta\sum_{\varepsilon'\in E,z'\in Z} v\left(a',\varepsilon';z',\lambda'\right)\pi\left(z',\varepsilon'|z,\varepsilon\right)\right\} \\
&\quad \text{s.t.} \\
c+a' &= w(z,\lambda)\,\varepsilon + R(z,\lambda)\,a \\
a' &\geq -\underline{a} \\
\lambda' &= \Psi\left(\lambda,z,z'\right)
\end{aligned}
$$

  where $\Psi(\lambda,z,z')$ is the law of motion of the endogenous aggregate state.

- Dependence on z' is inherited from π, but it does not apply to the marginal wrt wealth (a' is predetermined, thus only function of z).

- $\Psi(\lambda,z,z')$ is the new equilibrium object

Stochastic Recursive Equilibrium: Objects

Let the individual state space be $S \equiv A \times E$, where $A \subset \mathbb{R}$ is the asset space and E is a finite set of idiosyncratic states. Let Z be the finite aggregate shock space, and let Λ denote the set of probability measures on S.
For any aggregate state $(z, \lambda) \in Z \times \Lambda$, define $K(\lambda) = \int_S a \, d\lambda(a, \varepsilon)$, $\qquad H(z) = \sum_{\varepsilon \in E} \varepsilon \, \Pi_z(\varepsilon)$.

Definition. A *stochastic recursive competitive equilibrium* is a collection $\big(v, g, w, R, \Psi\big)$ such that:

1. **Household problem.** For all $(a, \varepsilon; z, \lambda)$,

$$v(a, \varepsilon; z, \lambda) = \max_{a' \geq \underline{a}} \left\{ u\big(w(z,\lambda)\varepsilon + R(z,\lambda)a - a'\big) + \beta \sum_{z' \in Z} \sum_{\varepsilon' \in E} v(a', \varepsilon'; z', \lambda') \, \pi(z', \varepsilon' | z, \varepsilon) \right\}$$

   subject to $\lambda' = \Psi(\lambda, z, z')$. The policy $g(a, \varepsilon; z, \lambda)$ attains the maximum.

2. **Firm optimality.**

$$w(z, \lambda) = z \, F_H\big(K(\lambda), H(z)\big), \qquad R(z, \lambda) = 1 + z \, F_K\big(K(\lambda), H(z)\big) - \delta.$$

SRCE: Transition Law, Consistency, and Rational Expectations

Fix a policy g and current aggregate state (z, λ). For each realization $z' \in Z$, define the **individual transition kernel**:

$$Q_g\big((a, \varepsilon; z, \lambda),\, z',\, B\big) = \sum_{\varepsilon' \in E} \mathbf{1}\big\{\big(g(a, \varepsilon; z, \lambda), \varepsilon'\big) \in B\big\} \Pr(\varepsilon' \mid \varepsilon, z, z'),$$

for all measurable $B \subseteq S$. The **induced law of motion of the distribution** is:

$$\Gamma_g(\lambda, z, z')(B) = \int_S Q_g\big((a, \varepsilon; z, \lambda),\, z',\, B\big)\, d\lambda(a, \varepsilon), \qquad B \subseteq S.$$

3. Consistency (Rational Expectations).

$$\Psi(\lambda, z, z') = \Gamma_g(\lambda, z, z') \qquad \forall (\lambda, z, z').$$

4. Market clearing.

$$K(\lambda) = \int_S a\, d\lambda(a, \varepsilon), \qquad H(z) = \sum_{\varepsilon \in E} \varepsilon\, \Pi_z(\varepsilon).$$

Why is the distribution a state variable?

- Consider the Euler equation associated to the problem above:

$$u_c\left(R\left(z,\lambda\right)a+w\left(z,\lambda\right)\varepsilon-g\left(a,\varepsilon;z,\lambda\right)\right)\geq$$
$$\beta\mathbb{E}\left[\mathbf{R}\left(\mathbf{z}',\lambda'\right)u_c\left(\mathbf{R}\left(\mathbf{z}',\lambda'\right)g\left(a,\varepsilon;z,\lambda\right)+\mathbf{w}\left(\mathbf{z}',\lambda'\right)\varepsilon'-g\left(g\left(a,\varepsilon;z,\lambda\right),\varepsilon';z',\lambda'\right)\right)\right]$$

- To solve for g, households need to forecast prices next period, and next period prices depend on λ'
- Agents need to know the equilibrium law of motion Ψ to forecast λ' given λ
- Ψ is a mapping from distributions into distributions: complicated
- Note: if R was exogenous (SOE), the wealth distribution would not be a state variable in this example

The Aggregate State as a Measure-Theoretic Object

Individual state space. Let

$$S \equiv A \times E,$$

where $A \subset \mathbb{R}$ is the asset space and E is a finite set of idiosyncratic states. Equip S with the product σ-algebra

$$\mathcal{S} \equiv \mathcal{B}(A) \otimes 2^E.$$

Cross-sectional distribution. At each date, the economy is summarized by a probability measure

$$\lambda : \mathcal{S} \to [0, 1],$$

satisfying

$$\lambda(S) = 1, \qquad \lambda\left(\bigcup_{m=1}^{\infty} B_m\right) = \sum_{m=1}^{\infty} \lambda(B_m) \quad \text{for disjoint } \{B_m\} \subset \mathcal{S}.$$

Interpretation: for each measurable set $B \in \mathcal{S}$,

$$\lambda(B) = \text{mass of agents with } (a, \varepsilon) \in B.$$

The Aggregate State as a Measure-Theoretic Object

Exact aggregate state. The exact recursive equilibrium state is

$$(z, \lambda) \in Z \times \mathcal{P}(S, \mathcal{S}),$$

where $\mathcal{P}(S, \mathcal{S})$ is the set of probability measures on $(S, \mathcal{S})$.

Why it matters for equilibrium. Under rational expectations, households must forecast future prices. Since future prices depend on future aggregates, and future aggregates depend on the future cross-sectional distribution, agents must forecast

$$\Psi(\lambda, z, z').$$

Why Is the Exact Distribution Infinite-Dimensional?

Finite-state benchmark. If both A and E were finite, then λ would be just a finite probability vector:

$$\lambda = (\lambda_1, \ldots, \lambda_N), \qquad \sum_{i=1}^{N} \lambda_i = 1.$$

But here the asset space is not finite. Because A is a continuum, for each idiosyncratic state ε, the distribution over assets is a whole cumulative distribution function

$$a \mapsto \lambda\big((-\infty, a] \times \{\varepsilon\}\big),$$

equivalently, a measure on Borel subsets of A.

Why Is the Exact Distribution Infinite-Dimensional?

Formal statement. Let $ca(S, \mathcal{S})$ denote the vector space of finite signed measures on $(S, \mathcal{S})$. Then

$$\mathcal{P}(S, \mathcal{S}) \subset ca(S, \mathcal{S}).$$

If A contains infinitely many distinct points and $E \neq \varnothing$, then $ca(S, \mathcal{S})$ is infinite-dimensional. Indeed, for any N, one can choose distinct $a_1, \ldots, a_N \in A$ and fixed $\bar{\varepsilon} \in E$, so that the Dirac measures

$$\delta_{(a_1, \bar{\varepsilon})}, \ldots, \delta_{(a_N, \bar{\varepsilon})}$$

are linearly independent.

Conclusion. The exact cross-sectional distribution λ is not a finite vector of coordinates. It is an infinite-dimensional state object.

One Discretized Distribution Is Easy to Store

Discretize the individual state space. Choose a partition $\{I_i\}_{i=1}^{n_a}$ of A, and write $E = \{\varepsilon_1, \ldots, \varepsilon_{n_e}\}$. Define the cells

$$C_{ij} \equiv I_i \times \{\varepsilon_j\}.$$

Cell-mass representation. A single cross-sectional distribution λ is approximated by

$$p_{ij} \equiv \lambda(C_{ij}), \qquad p_{ij} \geq 0, \qquad \sum_{i=1}^{n_a} \sum_{j=1}^{n_e} p_{ij} = 1.$$

Thus one discretized distribution is summarized by the vector

$$p = (p_{ij}) \in \Delta^{n_a n_e - 1},$$

where

$$\Delta^{n_a n_e - 1} = \left\{ x \in \mathbb{R}_+^{n_a n_e} : \sum_{i,j} x_{ij} = 1 \right\}.$$

One Discretized Distribution Is Easy to Store

Important distinction. This is not yet the curse of dimensionality. Storing one vector p is easy.

If the policy is known. Given a policy rule g, one can propagate the histogram forward and compute

$$p' = \Gamma_h(p, z, z')$$

by a non-stochastic simulation / histogram method.

Where the Curse of Dimensionality Actually Comes From

Representative-agent economy. Under rational expectations my saving decision is the aggregate:

$$K' = g(K, z).$$

So forecasting is low-dimensional.

Heterogeneous-agent economy. My saving decision does not determine the aggregate:

$$K' = \int g(a, \varepsilon; z, \lambda)\, d\lambda.$$

Under rational expectations, the household problem is $v(a, \varepsilon; z, \lambda)$, so the distribution is part of the state.

The Curse

After discretization. The numerical (approximated) representation of the value function becomes $v_h(a_i, \varepsilon_j; z, p)$, where $p \in \Delta^{n_a n_e - 1}$.

This is the curse. The problem is not representing one distribution. The problem is that dynamic programming requires approximating v_h and policy rules for *many possible* p's, i.e. over the whole simplex. If one used n_λ nodes per coordinate, the number of grid points would scale like $n_\lambda^{n_a n_e - 1}$.

Individual grid	Dim of simplex	Grid points
$n_a = 100,\ n_e = 2$	199	n_λ^{199}
$n_a = 500,\ n_e = 2$	999	n_λ^{999}
$n_a = 500,\ n_e = 7$	3499	n_λ^{3499}

Even $n_\lambda = 2$ gives $2^{199} \approx 8 \times 10^{59}$ in the coarsest case (!).

The issue is not computing one distribution — it's that rational expectations forces us to solve the model for every possible distribution.

The Krusell and Smith approach

- Krusell and Smith (KS, 1998) propose to approximate the distribution through a finite set of moments
- Let $\overline{m}$ be $(M \times 1)$ vector of moments of the wealth distribution, i.e. of the marginal of λ with respect to a
- Our new state vector has law of motion, in vector notation,

$$\overline{m}' = \Psi\left(z, \overline{m}\right)$$

- Note, that we loose the dependence on z' since we are only interested in the wealth distribution.

The Krusell and Smith approach

- To make this approach operational, one needs to: (i) choose M and (ii) specify a functional form for Ψ

- KS main finding: one obtains a very precise forecasting rule by simply setting $M = 1$ and specifying a law of motion of the form:

$$\log K' = b_z^0 + b_z^1 \log K$$

  or similarly in levels instead of logs.

- Near-aggregation holds because (i) the wealth-rich have close-to-linear saving rules, (ii) they matter a lot more in the determination of aggregate wealth, and (iii) aggregate shocks do not induce significant wealth redistribution across agents.

The Krusell and Smith algorithm

1. Specify a functional form for the law of motion, for example, linear or loglinear. In the linear case:
$$\overline{m}' = B_z^0 + B_z^1 \overline{m}$$
where B_z^0 is $M \times 1$ and B_z^1 is $M \times M$.
2. Guess the matrices of coefficients $\left\{B_z^0, B_z^1\right\}$
3. Specify how prices depend on $\overline{m}$. Since $\overline{m}_1 = K$:
$$\begin{aligned} R\left(z, \overline{m}\right) &= 1 + zF_K\left(\frac{\overline{m}_1}{H\left(z\right)}\right) - \delta \\ w\left(z, \overline{m}\right) &= zF_H\left(\frac{\overline{m}_1}{H\left(z\right)}\right) \end{aligned}$$

The Krusell and Smith algorithm

4. Solve the household problem and obtain the decision rule $g(a, \varepsilon; z, \overline{m})$ with standard methods.

   Note, you have additional state variables $\overline{m}$ on which you have to define a grid.

   At every point $(a, \varepsilon, z, \overline{m})$ on the grid solve for the choice $a' = a^*$ that satisfies the following EE:

$$u_c\left(R\left(z, \overline{m}\right) a + w\left(z, \overline{m}\right) \varepsilon - a^*\right) \geq$$
$$\beta \mathbb{E}\left[\mathbf{R}\left(\mathbf{z}', \mathbf{\Psi}\left(\mathbf{z}, \overline{\mathbf{m}}\right)\right) u_c\left(\mathbf{R}\left(\mathbf{z}', \mathbf{\Psi}\left(\mathbf{z}, \overline{\mathbf{m}}\right)\right) a^* + \mathbf{w}\left(\mathbf{z}', \mathbf{\Psi}\left(\mathbf{z}, \overline{\mathbf{m}}\right)\right) \varepsilon' - g\left(a^*, \varepsilon'; z', \Psi\left(z, \overline{m}\right)\right)\right)\right]$$

   where $\Psi(\cdot)$ is the forecasting function of the mean (first moment)

5. Simulate the economy for N individuals and T periods. In each period compute $\overline{m}_t$ from the cross-sectional distribution.

The Krusell and Smith algorithm

6. Run an OLS regression
$$\overline{m}_{t+1} = \widehat{B}_z^0 + \widehat{B}_z^1 \overline{m}_t$$
and estimate the coefficients $\left\{\widehat{B}_z^0, \widehat{B}_z^1\right\}$.

   Since the law of motion is time invariant, we can separate the dates t in the sample where the state is z_b from those where the state is z_g and run two distinct regressions.

7. If $\left\{B_z^0, B_z^1\right\} \neq \left\{\widehat{B}_z^0, \widehat{B}_z^1\right\}$, try a new guess and go back to step 1.

8. Continue until convergence: fixed point algorithm in $\left(B_z^0, B_z^1\right)$.

9. Assess whether the solution is accurate enough. If fit at the solution is not satisfactory add moments of the distribution or try a different functional form for the law of motion.

Step 5: Stochastic simulation

- This method approximates the continuum of agents with a large but finite number of agents and uses a random number generator to draw both the aggregate and the idiosyncratic shocks.

1. Simulate the economy for N individuals and T periods. For example, $N = 10,000$ and $T = 1,500$.

2. Draw first a random sequence for the aggregate shocks of length T. Next, one for the individual productivity shocks for each $i = 1, ..., N$, conditional on the time path for the aggregate shocks.

3. Initialize at $t = 0$ from the stationary distribution, for example.

4. Drop the first 500 periods when computing statistics.

Non-stochastic simulation

1. Draw a long series of aggregate shocks of length T.
2. Construct a fine grid over capital $[a_{\min}, a_{\max}]$, say, of $1,000$ points i.e. $J = 1,000$.
3. Initialize the distribution at $t = 0$ from the stationary distribution $\lambda_0(a, \varepsilon) = \lambda^*(a, \varepsilon)$.

Non-stochastic simulation

4. Suppose we are at a given date t of the simulation with aggregate states (m, z_t) and next period aggregate shock is z_{t+1}.

   Loop over the finer grid and for every ε and a_j on the finer grid for wealth compute $g\left(a_j, \varepsilon; z_t, \overline{m}_t\right)$.

   Identify the two adjacent grid points a_k and a_{k+1} that contain $g\left(a_j, \cdot\right)$. Then compute:

$$\begin{aligned}
\lambda_{t+1}\left(a_k, \varepsilon'\right) &= \sum_{\varepsilon \in E} \pi\left(\varepsilon', z_{t+1} | \varepsilon, z_t\right) \frac{a_{k+1} - g\left(a_j, \varepsilon; z_t, \overline{m}_t\right)}{a_{k+1} - a_k} \lambda_t\left(a_j, \varepsilon\right) \\
\lambda_{t+1}\left(a_{k+1}, \varepsilon'\right) &= \sum_{\varepsilon \in E} \pi\left(\varepsilon', z_{t+1} | \varepsilon, z_t\right) \frac{g\left(a_j, \varepsilon; z_t, \overline{m}_t\right) - a_k}{a_{k+1} - a_k} \lambda_t\left(a_j, \varepsilon\right)
\end{aligned}$$

5. Use these discretized distributions to compute the moments $\overline{m}_t$ period by period. This method is called the histogram method.

Wealth per capita of the unemployed

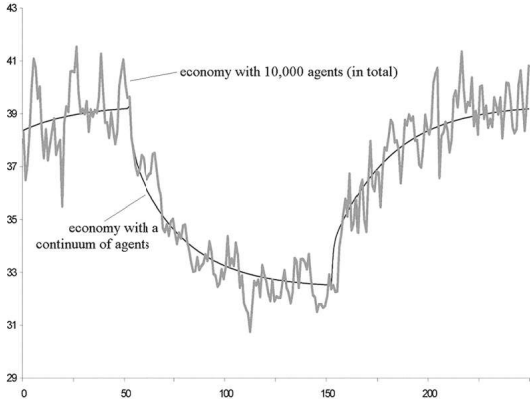

Figure 1 Simulated per capita capital of the unemployment. *Notes:* This graph plots the simulated aggregate capital stock of the unemployed using either a finite number (10,000) or a continuum of agents.

- Approximating a continuum with a finite number of agents introduces sampling variation in cross-sectional moments, which is especially large for small subpopulations (e.g., the unemployed).
- With a continuum of agents, a law of large numbers eliminates this variation (conditional on aggregate shocks), so the dynamics reflect the true equilibrium law of motion.

Wealth per capita of the unemployed

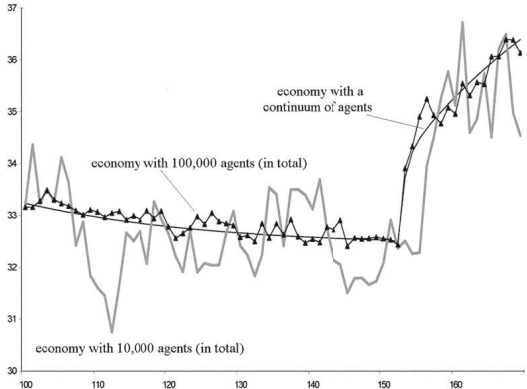

Figure 2 Simulated per capita capital of the unemployment. *Notes:* This graph plots the simulated aggregate capital stock of the unemployed using either a finite number (10,000) or a continuum of agents. It displays a subset of the observations shown in Figure 1.

- Even with $100{,}000$ agents, finite-agent simulation still generates noticeable sampling noise for the unemployed.
- The continuum path is computed by updating the cross-sectional distribution directly, not by simulating infinitely many agents.

Fraction of agents on the borrowing constraint

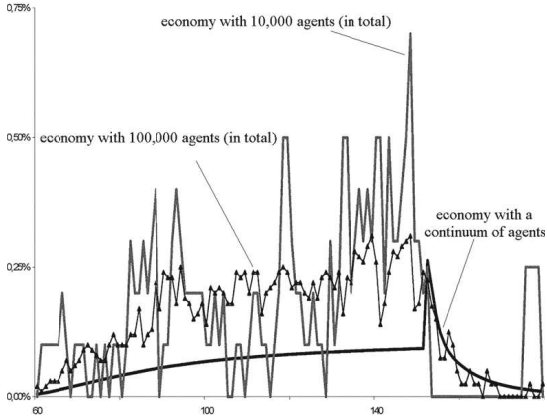

Figure 3 Simulated fraction of contrained agents. *Notes:* This graph plots the simulated fraction of unemployed agent at the borrowing constraint using either a finite number (10,000 or 100,000) or a continuum of agents.

- Sampling noise is *severe* for thin groups: even with $100{,}000$ agents, the fraction at the constraint is highly volatile.
- The continuum path reveals the true dynamics; finite simulations can completely mask economically meaningful patterns.

The Role of Non-Stochastic Simulation

What problem does the KS algorithm solve?

- The distribution λ is an infinite-dimensional state variable
- Households need to forecast prices, which depend on λ'
- KS replaces λ with a finite vector of moments m (e.g., $m = K$)
- This makes the household problem tractable: solve $v(a, \varepsilon; z, m)$

What problem does non-stochastic simulation solve?

- Given decision rules $g(a, \varepsilon; z, m)$, we need to simulate the economy forward (Step 5 of KS)
- Stochastic simulation (finite panel) introduces sampling noise, especially for small subpopulations (e.g., unemployed, constrained agents)
- Non-stochastic simulation updates λ_t directly on a grid using the histogram method $\Rightarrow$ no sampling error

Bottom line: They are *complements*, not substitutes.

- KS approximation $\Rightarrow$ solves the *dimensionality* problem (solving for g)
- Non-stochastic simulation $\Rightarrow$ solves the *accuracy* problem (simulating with g)

Why R^2 Is Misleading

KS accuracy diagnostic: simulate the economy, run OLS on the simulated data, and report R^2. KS find $R^2 > 0.99999$ and conclude that mean capital alone is sufficient to forecast aggregate dynamics.

The problem (Den Haan, 2010): the one-step-ahead forecast error is

$$u_{t+1} = K^*_{t+1} - \left(b^0_{z_t} + b^1_{z_t} K^*_t\right),$$

where K^*_t is the *true* simulated capital stock. The regression resets to the truth every period — errors never accumulate $\Rightarrow$ R^2 is artificially high.

Example. Let $b^1_{z_g}$ denote the slope coefficient in the good-state forecasting rule. Perturbing it away from its fitted value:

$b^1_{z_g}$	R^2	Std. Dev. of K
0.96404 (fitted)	0.99999729	0.0248
0.954187	0.99990000	0.0217
0.932479	0.99900000	0.0174
0.864099	0.99000000	0.0113

The Right Test: Let Errors Propagate

Key idea: instead of resetting to K_t^* each period, feed the model's own prediction back in.

Notation:

- K_t^*: true aggregate capital from the simulation
- $\hat{K}_t$: capital predicted by the approximate law of motion, initialized at $\hat{K}_0 = K_0^*$

The predicted path evolves recursively using *its own lagged value*:

$$\hat{K}_{t+1} = b_{z_t}^0 + b_{z_t}^1 \hat{K}_t.$$

A bad law of motion drifts away from K_t^* over time — errors compound.

What to report:

- Maximum absolute deviation: $\max_t |\hat{K}_t - K_t^*|$
- Plot K_t^* vs. $\hat{K}_t$ over the full simulation — visual inspection reveals *where* and *when* the approximation fails

Takeaway: Solutions with $R^2 > 0.999$ can exhibit persistent deviations of 1–2% of capital when errors are allowed to propagate. Always use this test.

Accuracy in Practice

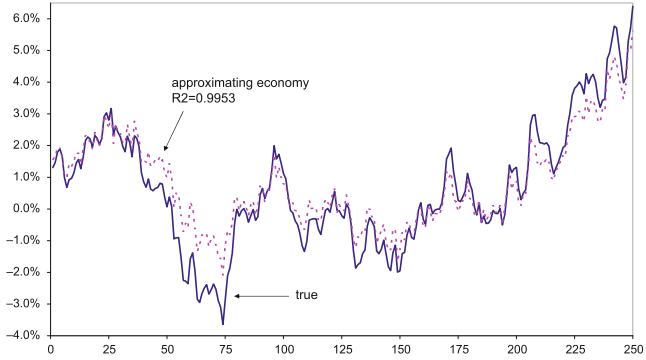

Figure 4 High R^2 and inaccurate law of motion. *Notes:* This graph plots the true aggregate capital stock and the one predicted by the approximate aggregate law of motion when the input of the approximation is the lagged value generated by the approximation and not the true lagged value (as is done when calculating the R^2).

Despite near-perfect R^2, the predicted path diverges by 1–2% of capital, persistently and correlated with recessions. If fit is unsatisfactory: add moments to m or change the functional form of Ψ.

Trivial market clearing

- When prices are determined by marginal product of the aggregate state variable (e.g., physical or human capital), then iterating over the law of motion for the aggregate state yields a solution where prices are consistent with market clearing
- Given a simulated history of $\{z_t\}$, and an initial condition K_t: (i) prices at t depend only on K_t which is known; (ii) individual saving decisions do not affect prices at t, but they aggregate into the same K_{t+1} predicted by the law of motion, and hence the asset market clears next period; (iii) next period prices equal again the marginal product of K_{t+1}.
- What makes market clearing trivial is that prices (wages and rates of return) only depend on a pre-determined variable K_t.

When Market Clearing Becomes Nontrivial

This breaks whenever a price must clear a market *today*:

1. **Endogenous labor supply.** The wage w_t depends on aggregate hours H_t, which is chosen at t. The distribution of wealth determines labor supply (wealth effects), so λ enters equilibrium wages directly.
2. **Risk-free bond (Huggett economy).** There is no physical capital. The interest rate r_t must clear the bond market *today*: $\int a'(a, \varepsilon;\, z, \lambda)\, d\lambda = 0$. The price r_t is a contemporaneous function of λ.
3. **Housing market.** The house price p_t^h clears a market where both demand and supply depend on the current distribution of wealth and housing.
4. **Heterogeneous firms.** Output prices, wages, or the interest rate may depend on the distribution of firm-level capital or productivity, requiring equilibrium conditions that cannot be solved forward.

In all these cases, solving for equilibrium prices *within* each period requires knowing λ — not just K. The KS moment approximation must now also deliver accurate price forecasts, not only K_{t+1}.

Readings

Core:

1. Krusell, P. and A. A. Smith, Jr. (1998). "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy*, 106(5), 867–896.
   doi:10.1086/250034
2. Algan, Y., O. Allais, W. J. Den Haan, and P. Rendahl (2014). "Solving and Simulating Models with Heterogeneous Agents and Aggregate Uncertainty." Ch. 6 in *Handbook of Computational Economics*, Vol. 3, Elsevier, 277–324.
   doi:10.1016/B978-0-444-52980-0.00006-2
3. Heer, B. and A. Maussner (2009). *Dynamic General Equilibrium Modeling*, 2nd ed., Springer. Chapters 8–9.
   doi:10.1007/978-3-540-85685-6

Supplementary (JEDC 2010 special issue):

4. Den Haan, W. J. and P. Rendahl (2010). "Solving the Incomplete Markets Model . . . Using Explicit Aggregation." *JEDC*, 34(1), 69–78. doi:10.1016/j.jedc.2009.07.009
5. Den Haan, W. J. (2010). "Comparison of Solutions to the Incomplete Markets Model . . ." *JEDC*, 34(1), 4–27. doi:10.1016/j.jedc.2008.12.010
6. Den Haan, W. J. (2010). "Assessing the Accuracy of the Aggregate Law of Motion . . ." *JEDC*, 34(1), 79–99.
   doi:10.1016/j.jedc.2008.12.009

Readings

Next lecture

1. Khan, A. and J. Thomas (2008). "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76(2), 395–436.
   doi:10.1111/j.1468-0262.2008.00837.x
2. Khan, A. and J. Thomas (2013). "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *JPE*, 121(6), 1055–1107.
   doi:10.1086/674142
3. Bloom, N. et al. (2018). "Really Uncertain Business Cycles," *Econometrica*, 86(3), 1031–1065.
   doi:10.3982/ECTA10927