Entrepreneurship and Financial Frictions

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This Lecture

- **Part I: The canonical occupational choice model**
  $\rightarrow$ Environment: Aiyagari + entrepreneurs + collateral constraints
  $\rightarrow$ Entrepreneur's static problem and the role of wealth
  $\rightarrow$ Occupational choice and decision rules
  $\rightarrow$ Equilibrium definition

- **Part II: Applications**
  $\rightarrow$ Quadrini (2000) — wealth concentration and social mobility
  $\rightarrow$ Buera and Shin (2013) — financial frictions and development dynamics
  $\rightarrow$ Moll (2014) — can self-financing undo misallocation?

Environment

- Continuum of agents, each characterized by state (s, ε, a)
  $\rightarrow s \in S$: labor productivity, Markov chain with transition Γ^s
  $\rightarrow \varepsilon \in \mathcal{E}$: entrepreneurial ability, Markov chain with transition Γ^ε, independent of s
  $\rightarrow a \geq 0$: wealth, endogenous
- Each period, agents choose an occupation $d \in \{0, 1\}$: worker or entrepreneur
- **Workers** supply labor with productivity s, earn wage ws
- **Entrepreneurs** are full-time managers. They operate a decreasing returns to scale technology $\varepsilon f(k, n)$, hiring capital k and labor n on the market
- Financial friction: entrepreneurs can borrow up to a fraction ϕ of their wealth

$$k - a \leq \phi\, a \qquad \Longleftrightarrow \qquad k \leq (1 + \phi)\, a$$

Worker's Problem

A worker earns wage income ws and gross return $(1+r)a$ on savings

$$V^w(s,\varepsilon,a) = \max_{c,\,a'} \;\; u(c) + \beta \sum_{s',\varepsilon'} \Gamma^s_{ss'}\,\Gamma^\varepsilon_{\varepsilon\varepsilon'} \max\Big\{V^w(s',\varepsilon',a'),\; V^e(s',\varepsilon',a')\Big\}$$

$$\text{s.t.} \quad c + a' = ws + (1+r)\,a, \qquad a' \geq 0$$

The $\max$ inside the expectation encodes the **occupational choice**: at state (s',ε',a') next period, the agent picks whichever occupation gives higher value. Define:

$$d(s',\varepsilon',a') = \begin{cases} 1 & \text{if } V^e(s',\varepsilon',a') > V^w(s',\varepsilon',a') \quad \text{(entrepreneur)} \\ 0 & \text{if } V^w(s',\varepsilon',a') \geq V^e(s',\varepsilon',a') \quad \text{(worker)} \end{cases}$$

This is a **policy function**, determined by the fixed point of the Bellman system

Entrepreneur's Problem

An entrepreneur earns profits $\pi(\varepsilon, a)$ from running a business

$$V^e(s, \varepsilon, a) = \max_{c,\, a'} \;\; u(c) + \beta \sum_{s', \varepsilon'} \Gamma^s_{ss'} \Gamma^\varepsilon_{\varepsilon\varepsilon'} \max \left\{ V^w(s', \varepsilon', a'),\; V^e(s', \varepsilon', a') \right\}$$

$$\text{s.t.} \quad c + a' = \pi(\varepsilon, a), \qquad a' \geq 0$$

- Same continuation value, same $d(s', \varepsilon', a')$, same savings problem
- The only difference from the worker is the income source: $\pi(\varepsilon, a)$ instead of $ws + (1 + r)a$
- $\pi(\varepsilon, a)$ does not depend on s – the entrepreneur does not supply labor. But s still enters through the continuation (it affects future V^w)

Together, the system (V^w, V^e) is solved by iterating on a pair of Bellman equations. The policy functions are: savings $g^w(s, \varepsilon, a)$, $g^e(\varepsilon, a)$, and occupational choice $d(s, \varepsilon, a)$

Entrepreneur's Profits

Given prices (w, r), the entrepreneur hires capital k and labor n to maximize:

$$\pi(\varepsilon, a) = \max_{k,\,n} \;\; \varepsilon\, f(k, n) + (1-\delta)k - (1+r)(k-a) - wn$$

$$\text{s.t.} \quad k - a \leq \phi\, a$$

- $\varepsilon f(k, n)$: output from DRS technology
- $(1+r)(k-a)$: cost of borrowing $k-a$ at rate r
- wn: labor cost (all labor hired on the market)
- Note: $\pi(\varepsilon, a)$ **already includes** $(1+r)a$ — the return on own funds. Compare with total worker income $ws + (1+r)a$
- Entrepreneur does not supply labor — full-time manager. The opportunity cost of entrepreneurship is the forgone wage ws

Unconstrained vs. Constrained Entrepreneur

Unconstrained ($\mu = 0$, constraint slack):
First-order conditions

$$\varepsilon f_k(k^*, n^*) = r + \delta, \qquad \varepsilon f_n(k^*, n^*) = w$$

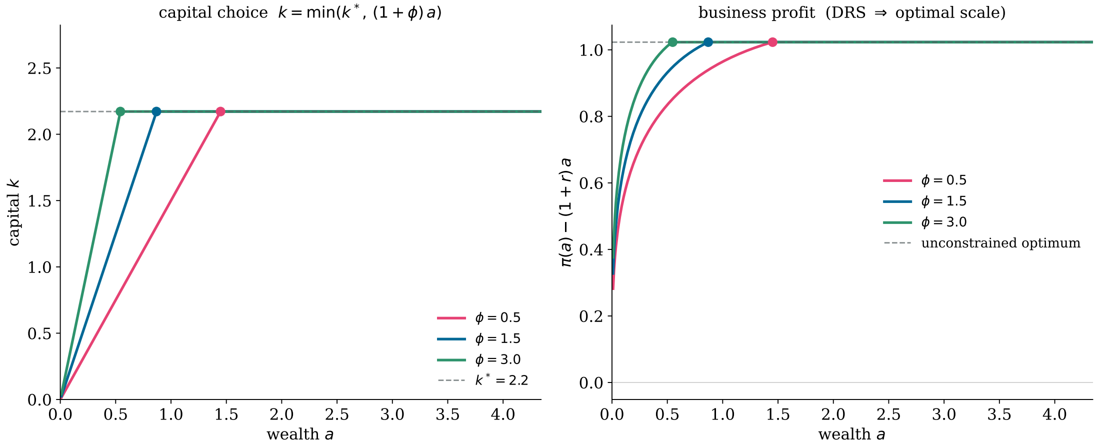
First-best scale $k^*(\varepsilon)$, $n^*(\varepsilon)$. Wealth a is irrelevant. Profit is $\pi^*(\varepsilon) + (1 + r)a$

Constrained ($\mu > 0$, constraint binds):
Capital is pinned by wealth

$$k = (1 + \phi)\, a$$

Labor adjusts to the constrained capital level: $\varepsilon f_n\big((1 + \phi)a,\ n\big) = w$
Both k and n are below first-best. The firm operates at inefficiently small scale. Profit is increasing and concave in a

Decision Rules: Capital and Business Profit



- Tighter ϕ: more wealth needed to reach efficient scale

Occupational Choice without Financial Frictions

Suppose $\phi \to \infty$ (no borrowing constraint)

- Wealth plays no role in the occupational choice
- Both entrepreneur and worker earn $(1 + r)a$ on savings, so it cancels. The comparison reduces to unconstrained business surplus vs. wage:

$$\text{entrepreneur iff} \quad \pi^*(\varepsilon) > ws$$

  where $\pi^*(\varepsilon) = \varepsilon f(k^*, n^*) - (r + \delta)k^* - wn^*$ is the pure business surplus
- There exists a threshold $\varepsilon^*(s)$: become entrepreneur iff $\varepsilon > \varepsilon^*(s)$
- Capital flows to equalize marginal products across all entrepreneurs
- No misallocation

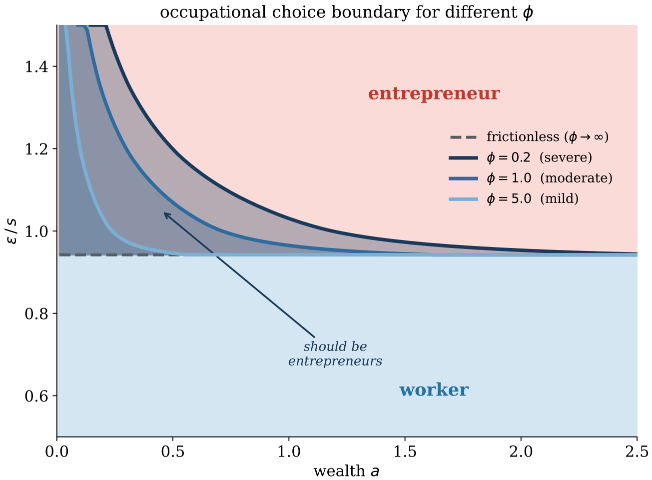
Occupational Choice with Financial Frictions

With $\phi < \infty$, wealth enters the occupational choice

- Wealthy agents with high ε: entrepreneurs (for sure)
- Poor agents with low ε: workers (for sure)
- High ε, low a: would like to be entrepreneurs, but the constraint limits their scale. If ε is persistent, they save aggressively to eventually enter
- Low ε, high a: lend their assets and work for a wage

The occupational choice boundary in (ε, a) space is a decreasing curve that converges to the frictionless threshold as $a \to \infty$

How Financial Frictions Distort Occupational Choice



- Without frictions: threshold is a **horizontal line** — only (ε, s) matter, wealth is irrelevant
- With frictions: boundary curves upward at low a. Tighter $\phi \Rightarrow$ larger distortion

Envelope Conditions

Using the envelope theorem on the value functions:

Worker:

$$V_a^w(s, \varepsilon, a) = (1 + r)\, u_c(c)$$

Entrepreneur:

$$V_a^e(s, \varepsilon, a) = \frac{\partial \pi}{\partial a}\, u_c(c) = \Big[(1 + r) + \mu\,(1 + \phi)\Big]\, u_c(c)$$

where $\mu \geq 0$ is the Lagrange multiplier on the collateral constraint

For a constrained entrepreneur ($\mu > 0$), an extra dollar of wealth has a double effect:

- It earns the market return $1 + r$, same as for a worker
- It relaxes the collateral constraint, unlocking $(1 + \phi)$ additional dollars of capital

The Euler Equation

The FOC for a' gives $u_c(c) = \beta\ \mathbb{E}\left[\frac{\partial}{\partial a'} \max\{V^w, V^e\}\right]$. Since the agent picks the best occupation at (s', ε', a'), the relevant envelope condition is:

$$u_c(c) = \beta\ \mathbb{E}\left[\max\left\{(1+r)\,u_c(c'^w),\ \ [(1+r) + \mu'(1+\phi)]\,u_c(c'^e)\right\}\right]$$

If the agent will be a constrained entrepreneur next period ($\mu' > 0$):

$$u_c(c) = \beta\ \mathbb{E}\left[\big(1 + r + \mu'(1+\phi)\big)\ u_c(c')\right]$$

Compare with the standard Aiyagari Euler equation: $u_c(c) = \beta(1+r)\ \mathbb{E}\left[u_c(c')\right]$

- The extra term $\mu'(1+\phi)$ is an endogenous return premium on saving
- Larger when the constraint is tighter, vanishes at the first-best scale
- This is **why constrained entrepreneurs save more and accumulate wealth faster**

Two Margins of Misallocation

The collateral constraint generates misallocation on two margins:

Extensive margin. Some high-ability agents are workers because they lack the wealth to operate at a profitable scale. Some low-ability agents are entrepreneurs because they are rich

Intensive margin. Among entrepreneurs, capital is not allocated to equalize marginal products. Constrained entrepreneurs have $\varepsilon f_k > r + \delta$: their capital is too low relative to their ability

Both margins reduce aggregate TFP below the first-best level

State Space and Transition Function

Individual state: $x = (s, \varepsilon, a) \in \mathcal{X} := S \times \mathcal{E} \times \mathcal{A}$, where $\mathcal{A} = [0, \bar{a}]$
Let $\mathcal{B}(\mathcal{X})$ denote the Borel σ-algebra on $\mathcal{X}$

Decision rules. Given prices (w, r), the household's problem yields:

- Savings policy: $g^w(s, \varepsilon, a)$ for workers, $g^e(\varepsilon, a)$ for entrepreneurs
- Occupational choice: $d(s, \varepsilon, a) \in \{0, 1\}$, where $d = 1$ means entrepreneur

Stationary Distributions

Let μ^w, μ^e be measures on $(\mathcal{X}, \mathcal{B}(\mathcal{X}))$ representing the mass of workers and entrepreneurs. Stationarity requires: for all $B = B_s \times B_\varepsilon \times B_a \in \mathcal{B}(\mathcal{X})$,

Workers tomorrow $=$ workers who stay $+$ entrepreneurs who switch:

$$\mu^{*w}(B) = \int_{\mathcal{X}} \Gamma^s(s, B_s)\, \Gamma^\varepsilon(\varepsilon, B_\varepsilon)\, \mathbf{1}\{g^w \in B_a\}\, \mathbf{1}\{d = 0\}\, \mu^{*w}(dx)$$

$$+ \int_{\mathcal{X}} \Gamma^s(s, B_s)\, \Gamma^\varepsilon(\varepsilon, B_\varepsilon)\, \mathbf{1}\{g^e \in B_a\}\, \mathbf{1}\{d = 0\}\, \mu^{*e}(dx)$$

Entrepreneurs tomorrow $=$ entrepreneurs who stay $+$ workers who switch:

$$\mu^{*e}(B) = \int_{\mathcal{X}} \Gamma^s(s, B_s)\, \Gamma^\varepsilon(\varepsilon, B_\varepsilon)\, \mathbf{1}\{g^e \in B_a\}\, \mathbf{1}\{d = 1\}\, \mu^{*e}(dx)$$

$$+ \int_{\mathcal{X}} \Gamma^s(s, B_s)\, \Gamma^\varepsilon(\varepsilon, B_\varepsilon)\, \mathbf{1}\{g^w \in B_a\}\, \mathbf{1}\{d = 1\}\, \mu^{*w}(dx)$$

Note: $d = d(s', \varepsilon', a')$ is the occupation chosen for *next period*, evaluated at the new state

Stationary Recursive Competitive Equilibrium

Definition. A stationary RCE consists of value functions V^w, V^e, policy functions g^w, g^e, d, k, n, prices w, r, and measures μ^{*w}, μ^{*e} on $(\mathcal{X}, \mathcal{B}(\mathcal{X}))$ such that:

1. **Household optimality.** Given (w, r), V^w and V^e solve the worker's and entrepreneur's Bellman equations with associated policies g^w, g^e, d
2. **Profit maximization.** $k(\varepsilon, a)$ and $n(\varepsilon, a)$ solve the entrepreneur's static problem
3. **Stationarity.** (μ^{*w}, μ^{*e}) satisfy the law of motion on the previous slide
4. **Capital market clears:**
$$\int k(\varepsilon, a)\ \mu^{*e}(dx) \ = \ \int g^w(s, \varepsilon, a)\ \mu^{*w}(dx) \ + \ \int g^e(\varepsilon, a)\ \mu^{*e}(dx)$$
5. **Labor market clears:** $\displaystyle \int n(\varepsilon, a)\ \mu^{*e}(dx) \ = \ \int s\ \mu^{*w}(dx)$

Smoothing the Occupational Choice: Gumbel Shocks

Add iid Gumbel$(0, \sigma)$ taste shocks ξ^w, ξ^e to each occupation. The ex-ante value function *before* the taste shock draw is the **log-sum-exp** formula:

$$\bar{V}(s, \varepsilon, a') = \sigma \log\Big(\exp\left(EV^w/\sigma\right) + \exp\left(EV^e/\sigma\right)\Big)$$

Choice probabilities (logistic):

$$p_e(s, \varepsilon, a) = \frac{1}{1 + \exp\left((V^w - V^e)/\sigma\right)}, \qquad p_w = 1 - p_e$$

- As $\sigma \to 0$: $\bar{V} \to \max(V^w, V^e)$ and $p_e \to \mathbf{1}\{V^e > V^w\}$ (sharp cutoff)
- As $\sigma \to \infty$: $p_e \to 1/2$ for all states (no selection)
- $\bar{V}$ contains **no max operator** $\Rightarrow$ differentiable. This is a numerical device, not a new economic friction

Occupational Choice: Sharp vs. Gumbel

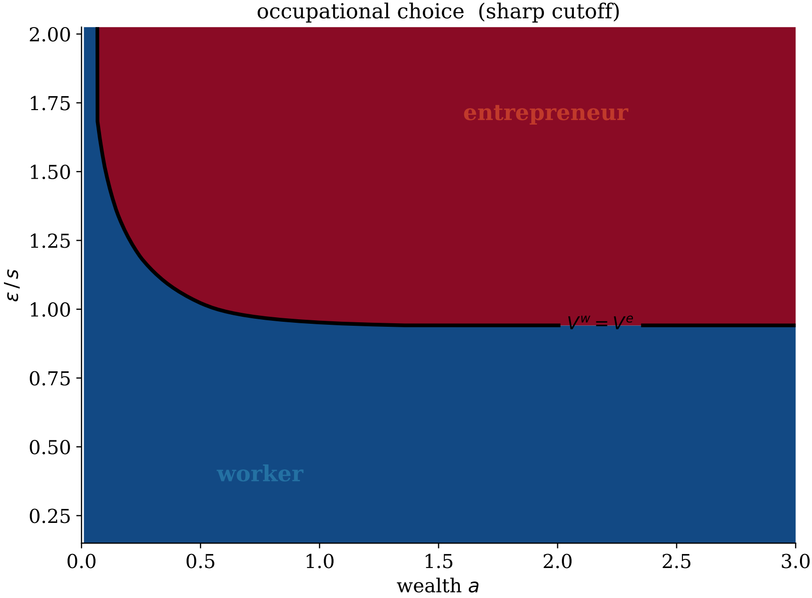
Without taste shocks ($\sigma = 0$): sharp boundary $V^w = V^e$ in $(\varepsilon,\ a)$ space (shown for a fixed s). The agent is either 100% worker or 100% entrepreneur. Decision rules jump discontinuously at the boundary

With Gumbel shocks ($\sigma > 0$): the boundary becomes a **smooth transition zone**. The probability p_e varies continuously from 0 (deep in the worker region) to 1 (deep in the entrepreneur region)

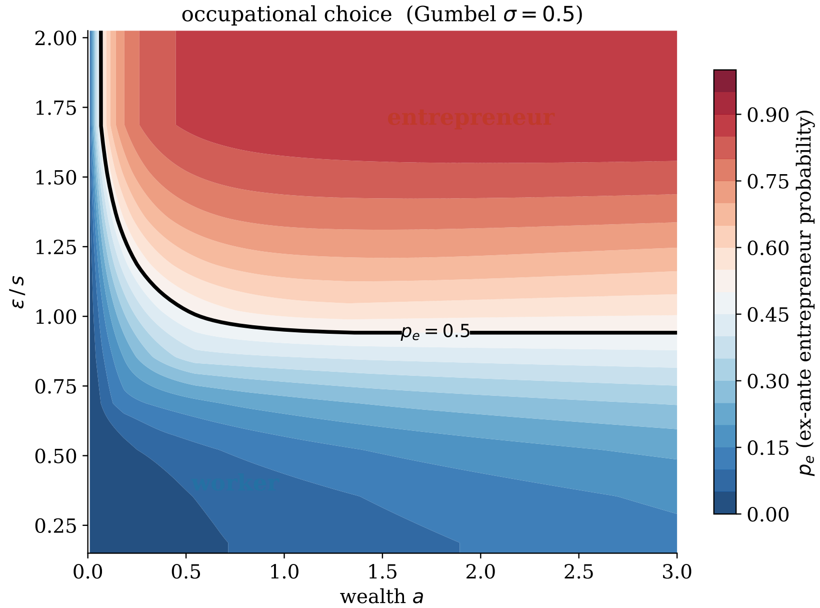
Implications for the stationary distribution:

- The transition function Q inherits the smoothness of p_e
- The stationary distribution μ^* is smoother and better behaved numerically
- For small σ: choice probabilities are nearly 0/1 away from the boundary, so the economics is essentially unchanged. The smoothing only matters *at the margin*

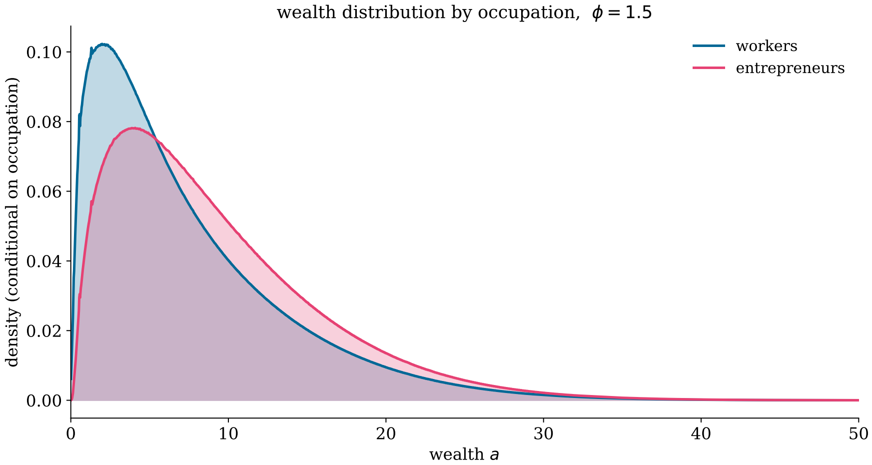
Occupational Choice: Sharp Cutoff



Occupational Choice: Gumbel Smoothing ($\sigma = 0.5$)

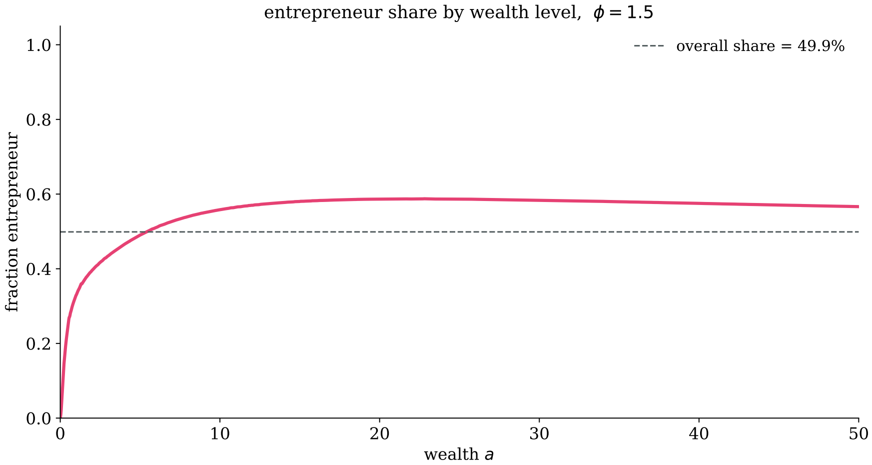


Stationary Distribution: Wealth by Occupation



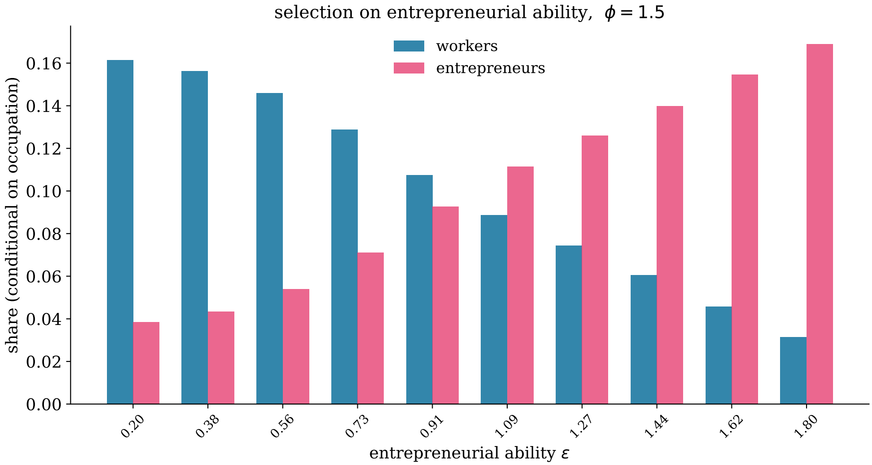
Entrepreneurs accumulate more wealth due to the double return on saving. The right tail of the wealth distribution is populated disproportionately by entrepreneurs

Stationary Distribution: Entrepreneur Share by Wealth



The share of entrepreneurs rises sharply with wealth. At the bottom: almost all workers.
This is the Quadrini/Cagetti–De Nardi fact

Stationary Distribution: Selection on Ability



Entrepreneurs are positively selected on entrepreneurial ability ε. Workers are the mirror image. This is the **comparative advantage** channel from the occupational choice

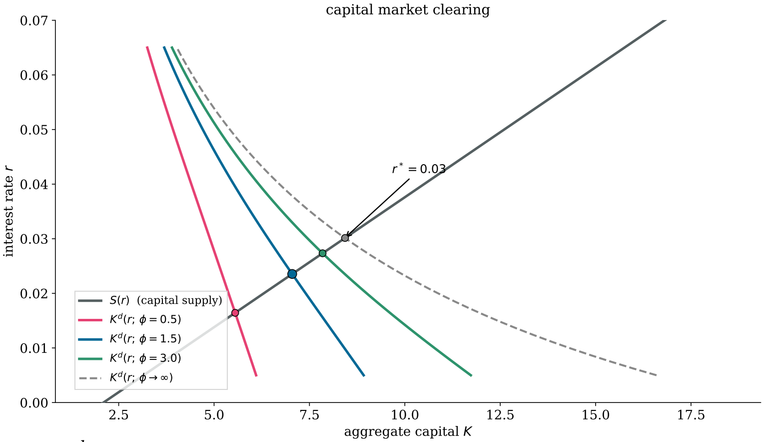
Role of the Interest Rate

The equilibrium interest rate r simultaneously:

- Clears the capital market: total savings $=$ total entrepreneurial demand for capital
- Determines the severity of the financial friction:
  $\rightarrow$ Higher r raises the cost of borrowing, reduces the scale of constrained entrepreneurs
  $\rightarrow$ This also makes it more attractive to be a lender (worker), shifting the occupational choice boundary
- Pins down the degree of misallocation in the cross-section

In the frictionless limit ($\phi \rightarrow \infty$): $r = \varepsilon f_k - \delta$ for all active entrepreneurs, no misallocation, wealth distribution has no aggregate consequences

Role of the Interest Rate



- Tighter ϕ shifts K^d left: constrained entrepreneurs deploy less capital at every r, depressing both equilibrium K and r^*
- The gap between frictioned and frictionless ($\phi \to \infty$) equilibria measures the aggregate cost of financial frictions

Part I Summary

Element	**Key result**
Occupational choice	Depends on (ε, s, a); only (ε, s) matter without frictions
Collateral constraint	$k \leq (1+\phi)a$ pins capital for low-wealth entrepreneurs
Envelope condition	$V_a^e = [(1+r) + \mu(1+\phi)]\, u_c$: double return on wealth
Euler equation	Endogenous return premium $\mu'(1+\phi)$ drives fast saving
Misallocation	Extensive (wrong agents) + intensive (wrong scale)

Part II: Applications

Quadrini (2000): Empirical Question

Why is wealth so much more concentrated than income?

- Top 1% of families own 30% of wealth (PSID) but earn only 7–8% of income
- Entrepreneurs are $\approx$10% of population but hold $\approx$40% of wealth
- Median entrepreneur wealth $\approx 5\times$ median worker wealth
- Standard Aiyagari model with labor income risk cannot generate this: the precautionary motive is too weak at high wealth (buffer-stock saving flattens out)

Second question: Entrepreneurs experience higher upward **wealth mobility** than workers (PSID transition matrices). What explains this?

Quadrini (2000): Model

Same core as the canonical model — but with different modeling choices

Infinitely-lived dynasties. State: $(\varepsilon, a, k, \eta)$ — labor ability, wealth, current project, productivity shock. Two sectors: corporate (CRS) and noncorporate (entrepreneurial)

Noncorporate technology:

$$y = \eta^{\nu}\, k^{\nu}\, n^{1-\nu}, \qquad 0 < \nu < 1$$

where $k \in K = \{0, k_1, \ldots, k_{N_k}\}$ is **indivisible** project capital, n is hired labor, η is an idiosyncratic shock

Departures from canonical model:

- Project k is discrete (choose *which* project, not how much capital)
- Intermediation cost ϕ on borrowing, not a hard collateral constraint
- Entrepreneur also earns wage εw (additional activity, not substitute)
- Learning by doing: $P_k(\kappa)$ depends on current project k

Quadrini (2000): Financial Frictions and Learning

Cost of capital depends on internal vs. external finance:

$$r = \begin{cases} r_D & \text{if } k \leq a \;\; \text{(self-financed)} \\ r_D + \phi \dfrac{k-a}{k} & \text{if } k > a \;\; \text{(debt-financed)} \end{cases}$$

$\Rightarrow$ Profit $\pi(a,k,\eta)$ is increasing in a: wealthier entrepreneurs borrow less, pay lower intermediation costs. Borrowing limited by repayment under worst $\eta_{\min}$

Learning. Each period, household draws new idea $\kappa \sim P_k(\kappa)$. Running project k increases the chance of getting $\kappa > k$

Budget constraint:

$$c = a(1+r_D) + \pi(a,k,\eta) + \varepsilon w - a'$$

Entrepreneur earns wage εw *plus* business profit π. Chooses $k' \in \{k, \kappa\}$ and savings a'

Quadrini (2000): Mechanism

Three forces generate high saving rates among entrepreneurs:

1. **Cost of external finance.** If $a < k$, the entrepreneur pays $r_D + \phi\frac{k-a}{k}$ on capital. An extra dollar of wealth reduces costly borrowing $\Rightarrow$ marginal return on saving exceeds r_D by ϕ/k. This is analogous to the "double return" in the canonical model
2. **Entrepreneurial risk.** Business income depends on the idiosyncratic shock η, which is uninsurable. Entrepreneurs face greater financial risk than workers $\Rightarrow$ larger precautionary buffer
3. **Entry and growth motive.** Workers save to accumulate the capital k_1 needed to implement their first project. Active entrepreneurs save to upgrade: running project k generates better ideas $\kappa > k$ (learning), but larger projects require more capital

All three forces push the wealth distribution rightward for entrepreneurs. Workers face only standard precautionary saving from ε risk, which flattens at high a

Quadrini (2000): Findings

- Wealth Gini: model ≈ 0.76 vs. data ≈ 0.78. Without entrepreneurship: Gini ≈ 0.50
- Entrepreneurs hold $\approx$40% of wealth in the model, matching the data
- Model generates higher upward wealth mobility among entrepreneurs: transition matrices line up with PSID
- Wealth concentration is driven by endogenous heterogeneity in returns, not income risk. This is the key departure from the standard incomplete markets literature

Limitation. Project size k is exogenous and indivisible — no continuous capital choice. The firm size distribution is driven by the idea process $P_k(\kappa)$, not by optimal investment

Extension: Cagetti and De Nardi (2006) add endogenous k, life cycle, and voluntary bequests. Match the wealth distribution more precisely and study estate tax policy

Buera–Shin (2013): Empirical Question

Why do miracle economies converge so slowly to their new steady state?
Seven miracle economies (China, Japan, Korea, Malaysia, Singapore, Taiwan, Thailand): all follow large-scale reforms in 1949–1983

- Half-life of capital stock in data: $\geq$15 years. Neoclassical model predicts $<$6 years
- TFP rises gradually for 15–20 years post-reform. Neoclassical model: TFP is constant
- Investment-to-output ratio is **hump-shaped**: starts low, peaks 5–10 years after reform. Neoclassical model: monotonically decreasing
- Financial markets remain underdeveloped throughout (private credit/GDP < 0.6 for 20 years)

Buera–Shin (2013): Empirical Facts

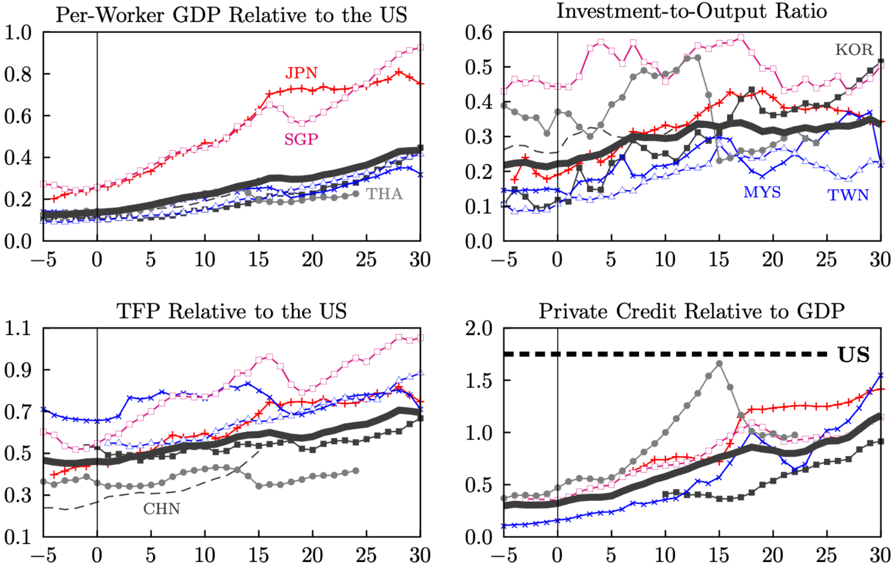


Fig. 1: Transitional Dynamics from the Economic Miracles

Buera–Shin (2013): Model

Occupational choice model. State: (e, a) – entrepreneurial ability and wealth

Technology. Entrepreneur with ability e hires k and l:

$$f(e, k, l) = e\big(k^{\alpha} l^{1-\alpha}\big)^{1-\nu}$$

where $1 - \nu$ is the span-of-control parameter (DRS). Collateral constraint: $k \leq \lambda a$

Profit:

$$\pi(e, a;\ w, r) = \max_{l,\, k \leq \lambda a} \Big\{ f(e, k, l) - wl - (\delta + r)k \Big\}$$

Agent becomes entrepreneur if $\pi(e, a) > w$. Threshold wealth $\underline{a}(e)$ solves $\pi(e, \underline{a}; w, r) = w$

Budget constraint (no borrowing for consumption, $a \geq 0$):

$$c + a_{t+1} \leq \max\{w,\ \pi(e, a)\} + (1 + r)a_t$$

Buera–Shin (2013): Pre-reform and Reform

Prereform steady state. Entrepreneurs face idiosyncratic distortions τ_i on output:

$$\pi_i^{\text{pre}} = (1 - \tau_i)\, f(e_i, k_i, l_i) - wl_i - (\delta + r)k_i$$

- $\tau_i > 0$: implicit tax on entrepreneur i (reduces profit)
- $\tau_i < 0$: subsidy (increases profit)

These wedges stand in for industrial policy, protectionism, entry barriers, size-dependent regulations (Restuccia–Rogerson, Hsieh–Klenow style)

$\Rightarrow$ Subsidized low-e entrepreneurs are artificially profitable $\Rightarrow$ wealthy
$\Rightarrow$ Taxed high-e entrepreneurs are artificially unprofitable $\Rightarrow$ poor

Reform $=$ **once-and-for-all** $\tau_i \to 0$**.** Financial frictions (λ) remain unchanged

$\Rightarrow$ Capital must flow to high-e agents. But they are poor and collateral-constrained $\Rightarrow$ must **save their way** to efficient scale. The joint distribution $G_t(e, a)$ is a slow-moving state variable

Buera–Shin (2013): Transition Dynamics

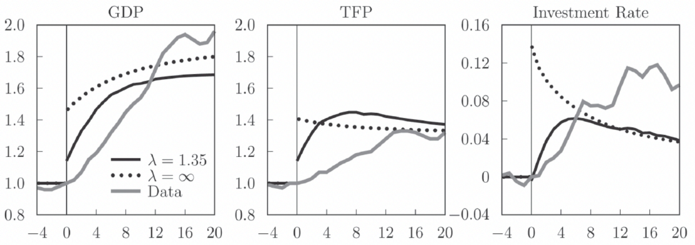


FIG. 3.—Benchmark transition: GDP, TFP, and investment rate. The black solid lines are the transitions from the benchmark exercise. For comparison, the transitional dynamics of a comparable neoclassical model are shown by the dotted lines, and the average of the economic miracles from Section I is represented by the gray lines (see App. B). The GDP and TFP series are normalized by their respective prereform values. Investment rates are shown as deviations from their prereform levels. The horizontal axes show years, and year 0 corresponds to the reform date.

Three key features of the model transition:

- **Slow convergence:** half-life of K is 10.5 years vs. 5.5 in neoclassical
- **Endogenous TFP:** rises gradually as capital reallocates to high-e entrepreneurs
- **Hump-shaped investment:** starts low (talented agents are poor), peaks after $\sim$6 years

Buera–Shin (2013): Findings

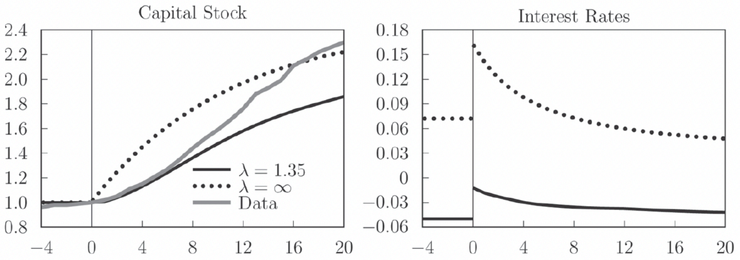


FIG. 4.—Benchmark transition: capital and interest rate. The black solid lines are the transitions from the benchmark exercise. Comparable neoclassical transition dynamics are shown by the dotted lines, and the evolution of the capital stock in the data, averaged across the miracle economies, is plotted by a gray solid line in the left panel (see App. B). The capital stock series are normalized by their respective prereform values. The horizontal axes show years, and year 0 corresponds to the reform date.

Decomposition:

- With $\lambda = \infty$ (perfect credit): model = neoclassical. Fast transition, no TFP dynamics
- Need *both* financial frictions ($\lambda < \infty$) *and* initial misallocation ($\tau \neq 0$) for slow, realistic dynamics
- Persistence ψ of e shocks matters: if $\psi = 1$ (permanent), frictions have zero SS effect but maximal transition effect. Foreshadows Moll (2014)

Moll (2014): Empirical Question

Can self-financing undo capital misallocation? Do financial frictions matter in the long run?

- Financial frictions cause misallocation $\Rightarrow$ lower TFP. But entrepreneurs can *save their way out*
- If self-financing works, frictions only matter during the transition
- If it doesn't, frictions cause permanent TFP losses
- A large quantitative literature gets very different answers: Buera–Shin find small SS losses, Midrigan–Xu find larger ones. Why?

Moll's answer: It depends on the persistence of productivity shocks. He develops a tractable model to prove this analytically

Moll (2014): Model

Continuous time. Entrepreneurs indexed by (z, a): productivity and wealth. Log preferences $\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c(t)\, dt$

Technology. Constant returns to scale: $y = f(z, k, \ell) = (zk)^\alpha \ell^{1-\alpha}$
Collateral constraint: $k \leq \lambda a, \;\; \lambda \geq 1$

Profit function:

$$\Pi(a, z) = \max_{k,\ell} \left\{ f(z, k, \ell) - w\ell - (r + \delta)k \;\; \text{s.t.} \;\; k \leq \lambda a \right\}$$

Lemma 1 (from the paper): CRS $\Rightarrow$ profits are *linear in wealth*:

$$\Pi(a, z) = \max\{z\pi - r - \delta, \; 0\} \cdot \lambda a, \qquad \pi \equiv \alpha \left(\frac{1 - \alpha}{w} \right)^{(1-\alpha)/\alpha}$$

Productivity cutoff for being active: $\underline{z}\,\pi = r + \delta$

Moll (2014): Linear Savings and Aggregation

CRS + log utility $\Rightarrow$ savings are linear in wealth (**Lemma 2**):

$$\dot{a} = s(z)\, a, \qquad s(z) = \lambda \max\{z\pi - r - \delta,\ 0\} + r - \rho$$

Define **wealth shares**: $\omega(z,t) \equiv \frac{1}{K(t)} \int_0^\infty a\, g_t(a,z)\, da$

Proposition 1: the economy is isomorphic to Solow with time-varying TFP:

$$Y = Z(t)\, K^\alpha L^{1-\alpha}, \qquad \dot{K} = \alpha Z K^\alpha L^{1-\alpha} - (\rho + \delta) K$$

where

$$Z(t) = \left(\frac{\int_{\underline{z}}^\infty z\, \omega(z,t)\, dz}{1 - \Omega(\underline{z}, t)} \right)^\alpha = \mathbb{E}_\omega[z \mid z \geq \underline{z}]^\alpha$$

TFP $=$ wealth-share-weighted average of productivities among active entrepreneurs

Moll (2014): The Role of Persistence

Productivity follows a diffusion: $dz = (1/\theta)\,\tilde{\mu}(z)\,dt + \sqrt{1/\theta}\,\tilde{\sigma}(z)\,dW$
θ governs persistence: $\theta \to \infty$ is fixed productivity, $\theta \to 0$ is iid

Two extremes:

- $\theta \to \infty$ (permanent z): highest-z entrepreneur saves fastest, eventually holds all wealth $\Rightarrow \omega(z) \to \mathbf{1}\{z = \bar{z}\} \Rightarrow$ TFP $\to \bar{z}^{\alpha}$ (first-best). Zero SS TFP loss
- $\theta \to 0$ (iid z): wealth and productivity are independent $\Rightarrow \omega(z) = \psi(z)$ (unconditional distribution). Self-financing is impossible. Maximum SS TFP loss

Proposition 4 (main result): For any ergodic process and any λ, steady-state TFP $Z(\theta)$ is **continuous and strictly increasing** in persistence θ, with

$$Z(0) = \mathbb{E}^{\alpha}_{z|z \geq \underline{z}} \qquad \text{and} \qquad \lim_{\theta \to \infty} Z(\theta) = \bar{z}^{\alpha}$$

Moll (2014): Wealth Shares and Self-Financing

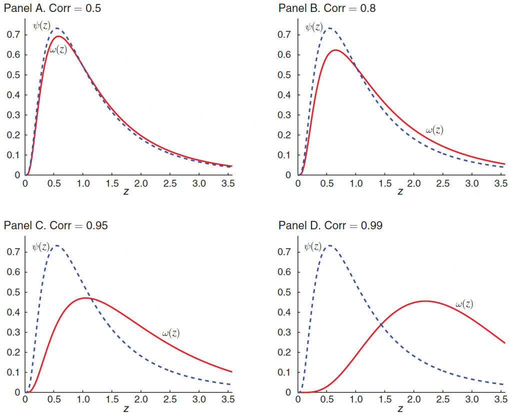


FIGURE 1. WEALTH SHARES AND AUTOCORRELATION

Notes: The dashed lines are the productivity distribution $\psi(z)$ from (27). The solid lines are the wealth shares $\omega(z)$: i.e., the solution to (22) for the stochastic process (26). As persistence θ (equivalently autocorrelation) increases, wealth becomes more concentrated with high-productivity entrepreneurs.

As persistence increases, self-financing shifts wealth toward high-z types: $\omega(z)$ first-order stochastically dominates $\psi(z)$ more and more. In the limit (permanent z): $\omega(z) \to \mathbf{1}\{z = \bar{z}\}$

Moll (2014): Findings

The persistence–misallocation trade-off:

High persistence (θ large): Self-financing works. Small SS TFP loss. But the transition to steady state is slow — wealth shares adjust gradually

Low persistence (θ small): Self-financing fails. Large SS TFP loss. But the transition is fast — shocks mix rapidly, distribution converges quickly

Reconciling the literature: Buera–Shin calibrate persistent shocks $\Rightarrow$ small SS losses, slow transitions. Midrigan–Xu calibrate less persistent $\Rightarrow$ larger SS losses. Not a contradiction: different persistence $\Rightarrow$ different margins matter

The persistence–misallocation trade-off.

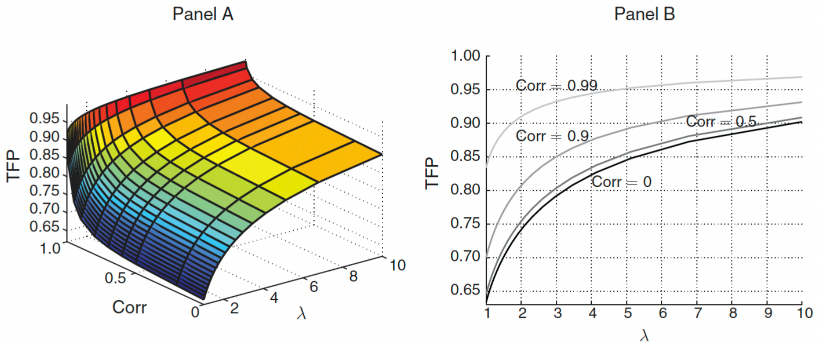


FIGURE 2. TFP AND AUTOCORRELATION

Notes: Panel B displays a cross-section of the three-dimensional graph in panel A. Again, note the sensitivity in the range corr $= 0.75$ to corr $= 1$. Parameters are $\alpha = 1/3$, $\rho = \delta = 0.05$, $\sigma\sqrt{-\log(0.85)} = 0.56$, and I vary corr $= \exp(-(1/\theta))$.

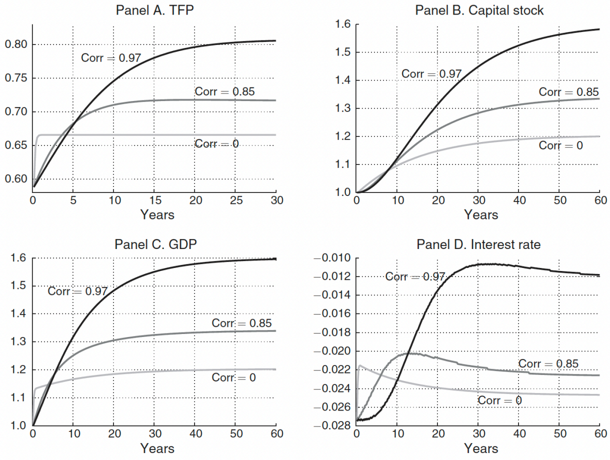


FIGURE 4. TRANSITION DYNAMICS FROM DISTORTED INITIAL WEALTH DISTRIBUTION

Notes: Parameter values are $\alpha = 1/3$, $\rho = \delta = 0.05$, and $\lambda = 1.2$, consistent with the external-finance-to-GDP ratio for India (see Table E1). For the benchmark exercise, I use $\text{corr} = \exp(-(1/\theta)) = 0.85$ and $\sigma\sqrt{1/\theta} = 0.56$. The lines for corr = 0 and corr = 0.97 vary θ while holding constant $\text{var}(\log z) = \sigma^2/2$. Initial wealth shares are given by (29) with $m = -0.5$.

Summary

Paper	Departure from canonical	Key insight
Quadrini	Discrete k, intermediation cost	Wealth concentration via returns
Buera–Shin	Two sectors + distortions τ_i	Wealth dist. is a slow state variable
Moll	CRS + log utility $\Rightarrow$ linear savings	Persistence $\leftrightarrow$ SS losses vs. speed

All build on the same core: occupational choice + collateral constraint. The friction creates a link between wealth and productive efficiency that generates rich cross-sectional and dynamic implications

Readings

Lecture 5 — Entrepreneurship and Financial Frictions

- Quadrini, V. (2000). “Entrepreneurship, Saving, and Social Mobility,” *RED*, 3(1), 1–40. doi
- Cagetti, M. and M. De Nardi (2006). “Entrepreneurship, Frictions, and Wealth,” *JPE*, 114(5), 835–870. doi
- Buera, F. and Y. Shin (2013). “Financial Frictions and the Persistence of History,” *JPE*, 121(2), 221–272. doi
- Moll, B. (2014). “Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?,” *AER*, 104(10), 3186–3221. doi
- Quadrini, V. (2009). “Entrepreneurship in Macroeconomics,” *Annals of Finance*, 5, 295–311. doi

Next Lecture

Lecture 6 — Heterogeneous Firms and Aggregate Dynamics

- Krusell, P. and A. Smith (1998). "Income and Wealth Heterogeneity in the Macroeconomy," *JPE*, 106(5), 867–896. doi
- Khan, A. and J. Thomas (2008). "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76(2), 395–436. doi
- Khan, A. and J. Thomas (2013). "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *JPE*, 121(6), 1055–1107. doi
- Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. Terry (2018). "Really Uncertain Business Cycles," *Econometrica*, 86(3), 1031–1065. doi

Exercise: Midterm Practice

Exercise: Setup (Moll, 2014 — Discrete Time with iid Shocks)

Discrete time, continuum of entrepreneurs with state (z, a). Productivity z drawn iid each period from $\psi(z)$ on $[1, \infty)$. Preferences: $\mathbb{E}_0 \sum \beta^t \log c_t$

Technology (CRS): $y = (zk)^\alpha \ell^{1-\alpha}$, capital depreciates at δ
Collateral constraint: $k \leq \lambda a$, $\lambda \geq 1$
Budget constraint:

$$a_{t+1} = (zk)^\alpha \ell^{1-\alpha} - w\ell - (r+\delta)k + (1+r)a - c$$

Mass L of hand-to-mouth workers supply labor inelastically

Assume Pareto productivity: $\Psi(z) = 1 - z^{-\eta}$, $\eta > 1$, $z \in [1, \infty)$

Exercise: Part (a) — Static Problem

Define the profit function $\Pi(a, z) = \max_{k,\ell}\{(zk)^\alpha \ell^{1-\alpha} - w\ell - (r+\delta)k \text{ s.t. } k \leq \lambda a\}$

(i) Solve for the optimal labor demand $\ell(a, z)$. Show that ℓ is proportional to zk

(ii) Substitute back and show that profits are **linear in wealth**:

$$\Pi(a, z) = \max\{z\pi - r - \delta,\ 0\} \cdot \lambda a, \qquad \pi \equiv \alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha}$$

(iii) Show that the optimal capital choice is a corner: $k = \lambda a$ if $z \geq \underline{z}$ and $k = 0$ otherwise. Find the **productivity cutoff** $\underline{z}$

(iv) Interpret: why does CRS imply that constrained entrepreneurs *always* want to invest as much as possible?

Exercise: Part (b) — Savings and Aggregation

(i) Using log utility and the linearity of the budget constraint in a, show that the optimal savings policy is $a_{t+1} = \beta\, A(z_t)\, a_t$ where $A(z) = \lambda \max\{z\pi - r - \delta, 0\} + 1 + r$
Hint: guess $v(a, z) = V(z) + B \log a$ *and verify*

(ii) Define wealth shares $\omega(z) \equiv \frac{1}{K} \int a\, g(a, z)\, da$. Explain why iid shocks imply $\omega(z) = \psi(z)$ for all t. Why does this fail with persistent shocks?

(iii) Using capital market clearing $\int k\, dG = \int a\, dG$ and labor market clearing, show the economy aggregates to:

$$Y = ZK^{\alpha}L^{1-\alpha}, \qquad K_{t+1} = \beta[\alpha Z K_t^{\alpha} L^{1-\alpha} + (1-\delta)K_t]$$

where $Z = \mathbb{E}^{\alpha}_{z|z\geq \underline{z}}$ and the cutoff satisfies $\lambda(1 - \Psi(\underline{z})) = 1$

Exercise: Part (c) — Pareto Productivity

Now assume $\Psi(z) = 1 - z^{-\eta}$, $\eta > 1$

(i) Show that the productivity cutoff is $\underline{z} = \lambda^{1/\eta}$

(ii) Compute $\mathbb{E}_{z|z \geq \underline{z}}$ for the Pareto distribution. Show that TFP is:

$$Z = \left(\frac{\eta}{\eta - 1} \, \lambda^{1/\eta} \right)^{\alpha}$$

(iii) Compute the elasticity of TFP with respect to credit quality: $\dfrac{\partial \log Z}{\partial \log \lambda} = \dfrac{\alpha}{\eta}$

Interpret: when are TFP losses from financial frictions large?

(iv) Compute the steady-state capital stock K^* and output Y^*. Show that the capital-to-output ratio $K/Y = \alpha\beta/(1 - \beta(1 - \delta))$ is **independent** of λ. Interpret

(v) Calculate the TFP loss from shutting down credit markets ($\lambda = 1$) relative to good credit ($\lambda = 10$) for $\eta \in \{1.1,\ 3,\ 10\}$ and $\alpha = 0.3$. Comment on the range of results

Exercise: Part (d) — Discussion

(i) In this model with iid shocks, TFP is constant over time. Explain intuitively why self-financing cannot improve the allocation, even though high-z entrepreneurs save more

(ii) What would change if productivity were persistent (e.g. z follows an AR(1))? Would you expect TFP losses to be larger or smaller? Would TFP be constant or time-varying during transitions?

(iii) Moll's main result (Proposition 4) says steady-state TFP is *strictly increasing* in persistence θ. Reconcile this with your answer to (ii): how can higher persistence mean *lower* steady-state TFP losses but *slower* transitions?

(iv) This model has no occupational choice — everyone is an entrepreneur. How would adding a worker option (as in the canonical model from Part I) change the expression for TFP? Which additional margin of misallocation would appear?