

Firm Dynamics

Entry, Exit, and Misallocation

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ECO2101 Lecture 4 | March 11, 2026

This Lecture

- [Part I: Hopenhayn \(1992\)](#) — entry, exit and long-run equilibrium
 - Model exposition
 - Sketch of computational procedure
 - Simplified static version for intuition
 - Computational details and practicalities
- [Part II: Hopenhayn and Rogerson \(1993\)](#) — nonconvex adjustment costs
 - Firm's lagged employment as an endogenous state variable
 - Inaction regions, misallocation, and aggregate consequences

Hopenhayn Model

- Workhorse model of industry dynamics
- **Steady-state model**: firms enter, grow and decline, and exit, but overall distribution of firms is unchanging
- Endogenous stationary distribution of firm-size etc, straightforward comparative statics

Key Elements

- Continuum of firms, no strategic interactions
- Produce with [decreasing returns to scale](#), so well-defined size
- Idiosyncratic risk: firm-level productivities follow a [Markov process](#)
- No aggregate risk
- Fixed cost to enter, fixed cost to operate each period

Setup

- Time $t = 0, 1, 2, \dots$
- Output and input prices p and w taken as given
- Output y produced with labor n given productivity z

$$y = zF(n)$$

- Static profits

$$\pi(z) = \max_{n \geq 0} [p z F(n) - w n - k]$$

where $k > 0$ is per-period fixed cost of operating

- Let $n(z)$ and $y(z)$ denote the associated optimal policies

Setup (cont.)

- Productivity

- $n(z)$, $y(z)$, $\pi(z)$ are all strictly increasing in productivity z
- Markov process for z with transition density $f(z' | z)$
- Entrants draw initial productivity z_0 from separate distribution $g(z)$, pay sunk cost $k_e > 0$ to do so

- Timing within a period

- Incumbents decide to stay or exit, entrants decide to enter or not
- Incumbents that stay pay k , entrants pay k_e
- After paying k or k_e , operating firms learn their productivity draws

Market Clearing

- Can choose either p or w as numeraire. We will choose $w = 1$
- Two key aggregate variables: (i) the price p , and (ii) the cross-sectional distribution of productivity types $\mu(z)$
- Industry supply curve, endogenous

$$Y(p) = \int y(z; p) \mu(z) dz$$

- Market clears when

$$Y(p) = D(p)$$

for some given demand curve $D(p)$

Incumbent's Problem

- Let $v(z; p)$ denote the value of incumbency to a firm with current productivity z given price p
- Bellman equation for an incumbent firm

$$v(z; p) = \pi(z, p) + \beta \max \left\{ 0, \int v(z'; p) f(z' | z) dz' \right\}$$

- An exit threshold $z^*(p)$ such that firm exits if $z < z^*(p)$, solves

$$\int v(z'; p) f(z' | z^*) dz' = 0$$

(for interior cases)

Entrant's Problem

- Potential entrants are ex ante identical
- Pay $k_e > 0$ to enter, initial draw from $g(z)$ if they do
- Start producing next period
- Let $m \geq 0$ denote the mass of entrants, free entry condition

$$\beta \int v(z; p) g(z) dz \leq k_e$$

with strict equality whenever $m > 0$

Stationary Equilibrium

A **stationary equilibrium** is a value function $v(z)$, output policy $y(z)$, cutoff productivity z^* , distribution $\mu(z)$, mass of entrants m and price p such that:

- (i) Taking p as given, $v(z)$, $y(z)$ and z^* solve the dynamic programming problem for an incumbent of type z
- (ii) The free-entry condition

$$\beta \int v(z) g(z) dz \leq k_e$$

is satisfied, with strict equality whenever $m > 0$

- (iii) The goods market clears

$$\int y(z) \mu(z) dz = D(p)$$

Stationary Equilibrium (cont.)

(iv) The distribution $\mu(z)$ is stationary

$$\mu(z') = \int \phi(z' | z) \mu(z) dz + m g(z')$$

where the conditional distribution $\phi(z' | z)$ is given by

$$\phi(z' | z) = f(z' | z) \times [z \geq z^*]$$

Stationary Distribution

- There is exit by firms with $z < z^*$ and entry $m g(z')$
- The distribution is stationary when

$$\mu(z') = \int \phi(z' | z) \mu(z) dz + m g(z')$$

- Suppose we discretize to a grid with n elements. Then this is a linear system of the form

$$\boldsymbol{\mu} = \boldsymbol{\Phi} \boldsymbol{\mu} + m \boldsymbol{g}$$

where $\boldsymbol{\Phi}$ is $n \times n$, $\boldsymbol{\mu}$ and $\boldsymbol{g}$ are $n \times 1$ and m is a scalar

Comparative Statics: Increase in k_e

Increase in entry cost k_e

- Increases expected discounted profits
- Decreases exit threshold z^*
 - Less selection, incumbents make more profits, more continue
 - Increases average age of firms
- Decreases mass of entrants m and entry/exit rate
- Increases price p

Ambiguous implications for firm-size distribution

- Price effect: higher k_e increases price p , hence incumbents increase output $y(z; p)$ and employment $n(z; p)$
- Selection effect: higher k_e reduces productivity threshold z^* , hence more incumbent firms are relatively-low productivity firms

Static Version for Intuition

- Once-and-for-all productivity draw $z \sim g(z)$, once-and-for-all endogenous exit. Production function $F(n) = n^\alpha$ for $0 < \alpha < 1$
- Static profit maximization problem

$$\pi(z; p) \equiv \max_{n \geq 0} [p z n^\alpha - n - k], \quad (w = 1 \text{ is numeraire})$$

- Implies employment, output

$$n(z; p) = (\alpha p z)^{\frac{1}{1-\alpha}}, \quad y(z; p) = z n(z; p)^\alpha$$

and profits

$$\pi(z; p) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} (p z)^{\frac{1}{1-\alpha}} - k$$

Exit and Entry Conditions

- Exit threshold z^* satisfies

$$\pi(z^*; p) = 0$$

such that firms immediately exit for all $z < z^*$

- Value of a firm given once-and-for-all choices

$$v(z; p) = \max \left\{ 0, \sum_{t=0}^{\infty} \beta^t \pi(z; p) \right\} = \max \left\{ 0, \frac{\pi(z; p)}{1 - \beta} \right\}$$

- Free entry condition

$$k_e = \beta \int v(z; p^*) g(z) dz = \beta \int_{z^*}^{\infty} \frac{\pi(z; p^*)}{1 - \beta} g(z) dz$$

Two conditions in two unknowns z^*, p^*

Implications for Selection

- Substituting the profit function into the free entry condition

$$(1 - \beta) k_e = \beta \int_{z^*}^{\infty} \left[(1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} (p^* z)^{\frac{1}{1-\alpha}} - k \right] g(z) dz$$

- Using the exit condition for z^* to eliminate p^* gives

$$(1 - \beta) \frac{k_e}{k} = \beta \int_{z^*}^{\infty} \left[\left(\frac{z}{z^*} \right)^{\frac{1}{1-\alpha}} - 1 \right] g(z) dz$$

- Increase in k_e (or decrease in k) **reduces cutoff z^*** and **increases p^*** , i.e., larger entry barriers weaken the selection effect and allow more unproductive firms to operate

Computing an Equilibrium (Sketch)

- **Step 1.** Guess output price p_0 . For this price, solve the incumbent's dynamic programming problem

$$v(z; p_0) = \pi(z; p_0) + \beta \max \left\{ 0, \int v(z'; p_0) f(z' | z) dz' \right\}$$

The solution also implies the optimal exit rule, i.e., the $z^*(p_0)$ that solves

$$\int v(z'; p_0) f(z' | z^*) dz' = 0$$

- **Step 2.** Check that this price p_0 satisfies the free-entry condition

$$\int v(z', p_0) g(z') dz' = k_e$$

If the LHS is too high then go back to Step 1, guess a new price $p_1 < p_0$. Continue until a price p^* is found that solves the free-entry condition

Computing an Equilibrium (cont.)

- **Step 3.** Guess a measure of entrants, m_0 . Given this, calculate the stationary distribution $\mu_0(z)$. This solves the linear system

$$\mu(z') = \int \phi(z' | z), \mu(z) dz + m g(z')$$

Observe that the RHS depends on the price found at Step 2 via the exit threshold $z^*(p^*)$

- **Step 4.** Given this $\mu_0(z)$, calculate the total industry supply and check the market clearing condition

$$Y = \int y(z, p^*) \mu_0(z) dz = D(p^*)$$

If the LHS is too low then go back to Step 3, guess new entrants $m_1 > m_0$. Continue until m^* is found that solves the market-clearing condition

Speeding Up the Last Step

- The stationary distribution is linearly homogeneous in m
- In terms of the discretized system above

$$\boldsymbol{\mu} = \boldsymbol{\Phi} \boldsymbol{\mu} + m \boldsymbol{g} \implies \boldsymbol{\mu} = m (\boldsymbol{I} - \boldsymbol{\Phi})^{-1} \boldsymbol{g}$$

where $\boldsymbol{I}$ is an identity matrix.

- Two implications
 - No need to use simulations to find stationary distribution $\boldsymbol{\mu}$, just set up coefficient matrix $\boldsymbol{\Phi}$ (implied by $z^*(p^*)$) and calculate directly
 - Only invert $(\boldsymbol{I} - \boldsymbol{\Phi})$ once, then just rescale by m

Solving the Hopenhayn Model

- Back to the dynamic version
- Suppose productivity follows *n*-state Markov chain on z_i with transition probabilities f_{ij}
- Given price p , value function is an n -vector with elements $v_i(p) \equiv v(z_i; p)$, $\pi_i(p) \equiv \pi(z_i; p)$, $y_i(p) \equiv y(z_i; p)$, etc
- Bellman equation for incumbent firm is then

$$v_i(p) = \pi_i(p) + \beta \max \left\{ 0, \sum_{j=1}^n v_j(p) f_{ij} \right\}$$

- Solve this by value function iteration

Free Entry

- Let $g_i = g(z_i)$ denote initial distribution over nodes z_i
- Given $v_i(p)$ that solves incumbent's problem, free entry condition is

$$v^e(p) \equiv \beta \sum_{i=1}^n v_i(p) g_i = k_e$$

whenever there is positive entry, $m > 0$ (for some parameter values, may have $m = 0$ in which case $v^e(p) < k_e$)

- Easy to show that $v^e(0) < 0$ and $v^e(p)$ monotone increasing in p , so interior solutions (with $m > 0$) can be found by [bisection](#)
- Intuitively, if $v^e(p) > k_e$ then reduce price to discourage entry but if $v^e(p) < k_e$ then increase price to encourage entry

Exit Decisions

- Given p^* , we know incumbent value function $v_i(p^*)$
- Exit threshold $z^*(p^*)$ then found from

$$z^*(p^*) = z_{i^*}, \quad i^* := \min_i \left[\sum_{j=1}^n v_j(p^*) f_{ij} \geq 0 \right]$$

All firms with $z_i < z^*(p^*)$ exit, all firms with $z_i \geq z^*(p^*)$ continue

- Collect exit decisions into a vector with elements

$$x_i(p^*) = \begin{cases} 1 & \text{if } z_i < z^*(p^*) \\ 0 & \text{if } z_i \geq z^*(p^*) \end{cases}$$

Stationary Distribution (Discrete)

- Let $\mu_i = \mu(z_i)$ denote mass of firms with productivity z_i
- Vector $\boldsymbol{\mu}$ solves

$$\boldsymbol{\mu} = \boldsymbol{\Phi}(p^*) \boldsymbol{\mu} + m \boldsymbol{g}$$

where $n \times n$ coefficient matrix $\boldsymbol{\Phi}(p^*)$ has elements

$$\Phi_{ij}(p^*) = (1 - x_j(p^*)) f_{ji}, \quad i, j = 1, \dots, n$$

- Stationary distribution

$$\boldsymbol{\mu} = m (\boldsymbol{I} - \boldsymbol{\Phi}(p^*))^{-1} \boldsymbol{g} \equiv \boldsymbol{\mu}(m, p^*)$$

for some m yet to be determined

Market Clearing (Discrete)

- Industry demand curve, exogenous $D(p)$
- Industry supply curve, endogenous

$$Y(m, p) = \sum_{i=1}^n y_i(p) \mu_i(m, p)$$

- We have solved for p^* from free entry condition (supposing $m > 0$). So now want to find measure of entrants m^* that solves

$$Y(m, p^*) = \sum_{i=1}^n y_i(p^*) \mu_i(m, p^*) = D(p^*)$$

- **Trick:** $\mu(m, p)$ is linear in m , so write $\mu(m, p) = m \times \mu(1, p)$ and solve for m^* as

$$m^* = \frac{D(p^*)}{Y(1, p^*)} = \frac{D(p^*)}{\sum_{i=1}^n y_i(p^*) \mu_i(1, p^*)}$$

Numerical Example

- Suppose preferences and technology

$$y = z n^{\alpha}, \quad D(p) = \bar{D}/p$$

- And that firm productivity follows AR(1) in logs

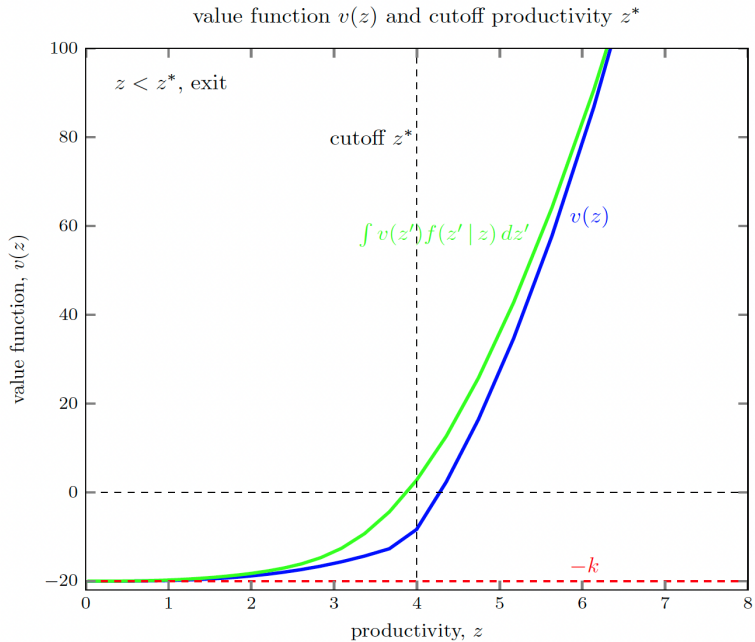
$$\log z_{t+1} = (1 - \rho) \log \bar{z} + \rho \log z_t + \varepsilon_{t+1}$$

- Parameter values (period length 5 years)

$$\alpha = 2/3, \quad \beta = 0.80, \quad k = 20, \quad k_e = 40$$

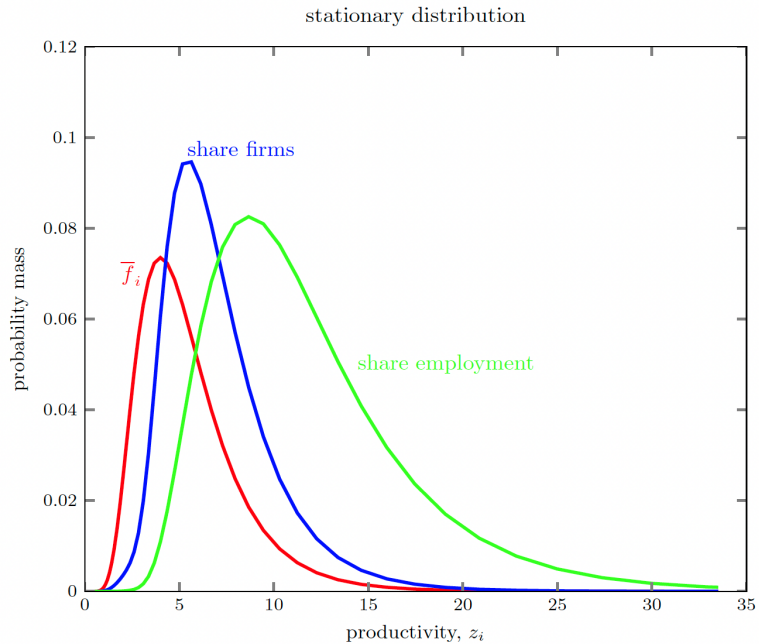
$$\log \bar{z} = 1.40, \quad \sigma = 0.20, \quad \rho = 0.9, \quad \bar{D} = 100$$

- Approximate AR(1) with Markov chain on 101 nodes



Value Function and Cutoff Productivity z^*

- **Exit rule.** The firm exits when the continuation value $\int v(z')f(z'|z) dz' < 0$ (green curve). The cutoff z^* is where the green curve crosses zero – not where $v(z)$ crosses zero.
- **Indifference at the cutoff.** At z^* , the firm is indifferent between staying and exiting. Current profits $\pi(z^*) < 0$ – the marginal firm operates at a loss. This is why $v(z^*)$ is negative even though the firm does not exit.
- **Low- z region.** For very low z , the value function flattens at $-k = -20$: revenue is negligible, so $v(z) \approx -k + \beta \cdot 0 = -k$.



Stationary Distribution

- **Underlying process.** The red curve $\bar{f}_i$ is the unconditional stationary distribution of the AR(1) productivity process — what the distribution would look like absent any entry or exit.
- **Selection effect.** Exit truncates the left tail: firms with $z < z^*$ are removed, so the equilibrium firm distribution (blue) is shifted to the right relative to $\bar{f}_i$.
- **Size-weighted reallocation.** Because employment $n(z) = (\alpha pz)^{1/(1-\alpha)}$ is convex in z , the employment-weighted distribution (green) has a much fatter right tail than the firm count distribution (blue) — most firms are small, but most workers are at a few highly productive firms.

Part I Summary

Concept	Key result
Optimal stopping	Firm exit: continue iff $z \geq z^*$
Free entry	Pins down price p^* via bisection
Stationary distribution	$\mu = m(I - \Phi)^{-1}g$, linear in m
Market clearing	Pins down m^* by rescaling
Comparative statics	Higher $k_e \Rightarrow$ lower z^* , higher p^*

Part II: Hopenhayn and Rogerson (1993)

Nonconvex adjustment costs, misallocation,
and aggregate consequences

Motivation

- Empirically: large labor market flows at the firm level (job creation and destruction)
- **Policy question**: what are the consequences of policies that make it costly for firms to adjust employment?
 - Firing taxes, severance mandates, employment protection legislation
- Key extension relative to Hopenhayn (1992): **nonconvex adjustment costs** make a firm's lagged employment n_{-1} an endogenous state variable
- Adjustment costs induce **misallocation** of resources across heterogeneous producers

General Equilibrium: Household Side

- Representative consumer with preferences

$$U(C, N) = \theta \log C - N, \quad \theta > 0$$

- Steady state with discount factor $\beta = 1/(1 + r)$
- Budget constraint: $pC \leq N + \Pi + T$, where $w = 1$ is numeraire, Π = aggregate profits, T = lump-sum transfers (tax revenues)
- First-order conditions imply:

$$C(p) = \theta/p, \quad N = \theta - \Pi - T$$

- Perfectly elastic labor supply – GE demand curve is $D(p) = \theta/p$

Firm's Problem with Adjustment Costs

- Output $y_t = z_t F(n_t)$, static profits now include adjustment costs:

$$p z F(n') - n' - H(n', n) - k$$

where $H(n', n)$ captures labor adjustment costs (in units of labor)

- Firing tax specification:

$$H(n', n) = \tau \times \max [0, n - n']$$

Firm pays τ per unit of labor *destroyed* — a tax on job destruction

- Note: H is **nonconvex** — introduces a kink at $n' = n$

Timing and State Space

- Incumbent begins period with state (z, n)
- Decides to **exit or stay**
 - If exit: pay $H(0, n) = \tau n$ this period, zero in future
 - If stay: choose n' optimally and then draw new $z' \sim f(z' | z)$
- Key difference from Hopenhayn (1992): state space is now (z, n) instead of just z
- Entrants start with $n = 0$ (no adjustment cost on entry)

Incumbent's Bellman Equation

- Value function $v(z, n; p)$ for a firm that has decided to operate and has just drawn z :

$$v(z, n; p) = \max_{n' \geq 0} \left\{ p z F(n') - n' - H(n', n) - k \right. \\ \left. + \beta \max \left[-H(0, n'), \int v(z', n'; p) f(z'|z) dz' \right] \right\}$$

- Optimal employment policy: $n' = \eta(z, n; p)$
- Optimal exit policy: $\chi(z, n; p) \in \{0, 1\}$
- Distribution $\mu(z, n)$ is now **two-dimensional**

Free Entry and Stationary Distribution

- Entrants begin with $n = 0$, free entry:

$$\beta f \int v(z, 0; p) g(z) dz \leq k_e, \quad \text{with equality if } m > 0$$

- Transition kernel:

$$\phi(z', n' | z, n) \equiv f(z'|z) \cdot \mathbf{1}[n' = \eta(z, n; p)] \cdot \mathbf{1}[\chi(z, n; p) = 0]$$

- Stationary distribution $\mu(z', n')$:

$$\mu(z', n') = \iint \phi(z', n' | z, n) \mu(z, n) dz dn + m g(z') \mathbf{1}[n' = 0]$$

- Same computational strategy: solve for p^* via free entry, then m^* via market clearing (using linearity of μ in m)

Aggregation

- Aggregate output

$$Y = \iint z F(\eta(z, n; p)) \mu(z, n) dz dn$$

- Aggregate employment (production workers + overhead)

$$N = \iint (\eta(z, n; p) + k) \mu(z, n) dz dn$$

- Aggregate profits

$$\Pi = \iint \pi(z, n; p) \mu(z, n) dz dn$$

- Aggregate firing costs

$$T = \iint H(\eta(z, n; p), n) \mu(z, n) dz dn$$

Market Clearing in GE

- Representative consumer's budget constraint

$$pC \leq N + \Pi + T$$

where T denotes revenues from adjustment costs, rebated lump-sum

- Goods market clearing:

$$Y = C(p) = \theta/p$$

- Labor market clearing:

$$N = \theta - \Pi - T$$

- So indeed: if goods market clears at price p , labor market also clears (Walras' law)

Optimal Employment: Inaction Regions

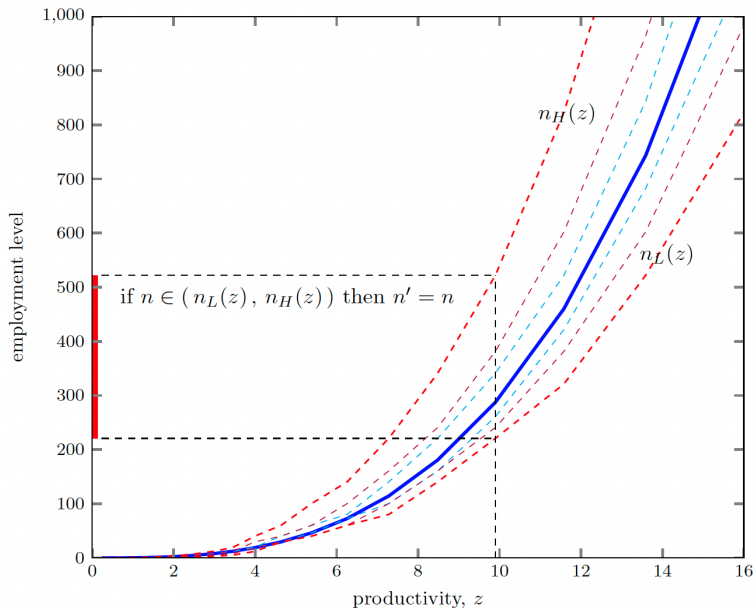
- If $\tau = 0$: employment is $n' = (\alpha z p)^{1/(1-\alpha)}$, independent of n
→ Log employment proportional to log productivity
- If $\tau > 0$: inaction region — for each z , there exist $n_L(z) < n_H(z)$ such that

$$\eta(z, n; p) = n \quad \text{whenever } n \in (n_L(z), n_H(z))$$

and resets to a value independent of n outside this region

- Higher $\tau \Rightarrow$ wider inaction region for each z
- Firms tolerate being at the “wrong” scale to avoid paying firing costs

employment inaction regions $n_L(z)$, $n_H(z)$



Employment Inaction Regions $n_L(z)$, $n_H(z)$

- **Frictionless benchmark.** The solid blue curve is the optimal employment policy $n(z) = (\alpha p z)^{1/(1-\alpha)}$ when $\tau = 0$. Each firm sets $\text{MPL} = 1/p$.
- **Inaction region.** With $\tau > 0$, if current employment $n \in (n_L(z), n_H(z))$ the firm does not adjust: $n' = n$. The dashed pairs show the boundaries for increasing values of τ — higher firing costs widen the band.
- **Misallocation.** Firms inside the band operate at the wrong scale. A firm near $n_H(z)$ has too many workers (MPL too low), a firm near $n_L(z)$ has too few (MPL too high). Neither adjusts, so marginal products are dispersed across firms and aggregate productivity falls.

Misallocation

- Without adjustment costs ($\tau = 0$): marginal product of labor equalized across all firms

$$\alpha z F'(\eta(z, n; p)) = \frac{1}{p} \quad \text{for all } z, n$$

Aggregate productivity $A = 1/(\alpha p)$ – efficient allocation

- With adjustment costs ($\tau > 0$): many firms have $\text{MPL} \neq 1/p$
 - Firms inside the inaction region operate at inefficient scale
 - Higher $\tau \Rightarrow$ wider inaction $\Rightarrow$ larger MPL deviations
 - Reduces aggregate productivity and output

Aggregate Effects of Firing Costs

	$\tau = 0$	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.5$
price	1.000	1.013	1.023	1.044
aggregate output	100.0	98.7	97.8	95.8
aggregate productivity	1.500	1.490	1.483	1.450
aggregate profit	20.2	22.6	24.5	25.9
firing costs / wage bill	0.000	0.017	0.028	0.042

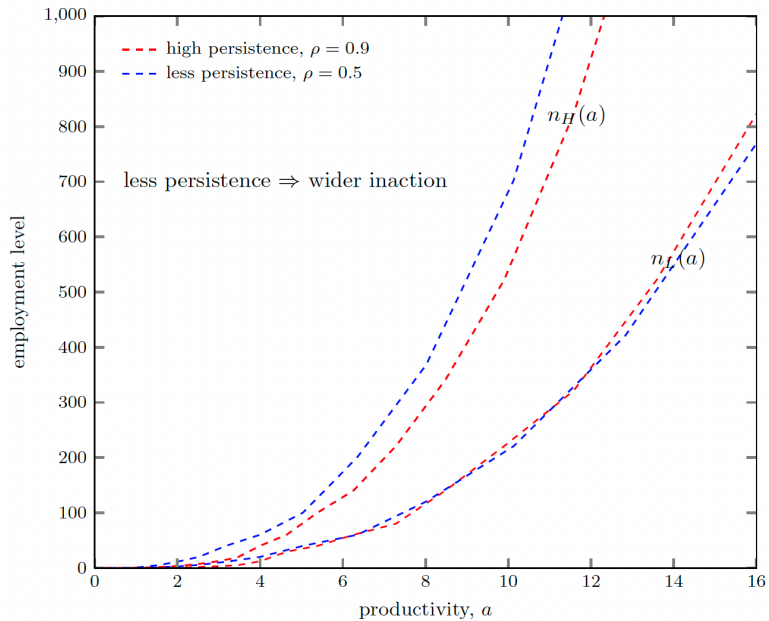
- As τ increases: output and productivity **fall**, price and profits **rise**
- Even moderate firing costs ($\tau = 0.1$, about 6 months' pay) generate nontrivial output losses

Role of Persistence ρ

- When shocks are very persistent (high ρ):
 - Efficient scale doesn't change often $\Rightarrow$ adjustment costs **less binding**
- When shocks are less persistent (low ρ):
 - Efficient scale changes frequently $\Rightarrow$ adjustment costs **more binding**
 - Wider inaction region, more misallocation
- Example ($\tau = 0.5$):

	$\rho = 0.9$	$\rho = 0.5$
aggregate output	95.8	86.1
aggregate productivity	1.450	1.385
firing costs / wage bill	0.042	0.074
entry/exit rate	0.219	0.068

inaction regions for $\rho = 0.5$ and $\rho = 0.9$, both with $\tau = 0.5$



Summary

Hopenhayn (1992)	Hopenhayn–Rogerson (1993)
State: z only	State: (z, n) – lagged employment
Static labor choice	Dynamic labor choice with adjustment costs
Exit threshold z^*	Exit threshold $z^*(n)$
MPL equalized	MPL dispersed $\Rightarrow$ misallocation
Selection via k_e, k	Selection + inaction regions via τ
$\mu(z)$ one-dimensional	$\mu(z, n)$ two-dimensional

Takeaway: even modest firing costs generate sizable output losses by preventing reallocation of labor to high-productivity firms

Readings

Lecture 4 – Firm Dynamics

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