Introduction to Modern Macroeconomics

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Course Overview

Tentative arc for the quarter:

- Lecture 1: Introduction—equilibrium, recursion, and why macro needs structure
- Lectures 2–3: Dynamic programming (theory and computation)
- Lectures 4–5: Recursive equilibrium applications (firm dynamics)
- Lectures 6–7: Firm dynamics with aggregate risk (Khan-Thomas, Ottonello-Winberry)
- Lectures 8–10: HANK and sequence space Jacobian methods

Textbooks (both free and open access):

- Azzimonti, Krusell, McKay & Mukoyama, *Macroeconomics*: `https://phdmacrobook.org`
- Sargent & Stachurski, *Dynamic Programming*: `https://dp.quantecon.org`

Roadmap: Equilibrium, Recursion, and Why Macro Needs Structure

1. What is a macroeconomic model?
   - $\rightarrow$ Environment $\rightarrow$ equilibrium mapping
2. Equilibrium as **consistency**
   - $\rightarrow$ Not stability, not efficiency, not uniqueness
3. One economy, three representations:
   - $\rightarrow$ Arrow–Debreu (complete plans at $t = 0$)
   - $\rightarrow$ Sequential markets (trade period by period)
   - $\rightarrow$ Recursive competitive equilibrium (functions + fixed point)
4. Two macro lessons:
   - $\rightarrow$ Aggregation breaks with heterogeneity
   - $\rightarrow$ Micro evidence misses the "intercept" $\Rightarrow$ general equilibrium matters

Goal: Unify the equilibrium concepts you have seen and connect them to modern macro practice.

Macroeconomic Model: Stochastic Dynamic Environment

A modern macro model specifies a **stochastic dynamic economy**.

Environment (Primitives)

- State space $\mathcal{S}$ and exogenous shock process $\{z_t\}$
- Law of motion for states (technology, shocks, institutions)
- Agents' preferences over stochastic sequences
- Feasibility constraints and market structure

Information and Expectations

- Agents observe information sets $\mathcal{I}_t$
- Expectations are formed conditional on $\mathcal{I}_t$

Decision Rules

- Agents choose policy functions mapping states and information into actions

One Economy, Three Representations

Same underlying economy. Different representations.

1. Arrow–Debreu (AD)

- Complete contingent plans chosen at $t = 0$
- Price functional over dated and state-contingent goods
- Equilibrium = optimality + feasibility + market clearing

2. Sequential Markets (SME)

- Trade occurs period by period
- Intertemporal optimization with asset trading
- Equivalent to AD under complete markets and standard no-arbitrage conditions

3. Recursive Competitive Equilibrium (RCE)

- Markov representation of SME
- Equilibrium objects are policy and pricing functions
- Aggregate law of motion imposed as a fixed point

Key message: AD = contingent sequences. SME = dynamic trading. RCE = fixed point in functions (Markov representation).

Markov Equilibrium

Let the exogenous environment follow a Markov process

$$z_{t+1} \sim \Pi(z_t).$$

An equilibrium is called **Markov** if:

- There exists a finite-dimensional state variable s_t such that

$$\mathbb{P}(s_{t+1} \mid h_t) = \mathbb{P}(s_{t+1} \mid s_t),$$

  where h_t is the full history.
- All equilibrium objects depend only on s_t:

$$c_t = c(s_t), \quad a_t = a(s_t), \quad p_t = p(s_t).$$

Interpretation: History matters only through the state vector.

Recursive Competitive Equilibrium (RCE)

Given a Markov environment with state s:

Recursive Competitive Equilibrium is a collection of functions:

$$\{V(s),\ g(s),\ p(s),\ G(s)\}$$

such that:

1. **Optimization:** Given $p(s)$ and $G(s)$, agents solve a dynamic program and choose $g(s)$.
2. **Firm optimality:** Prices $p(s)$ satisfy profit maximization.
3. **Consistency:** The aggregate law of motion $G(s)$ is implied by individual policies:

$$s' = G(s).$$

Equilibrium = fixed point in policy and pricing functions.

Roadmap

Understanding Equilibrium

The Neoclassical Growth Model

Adding Heterogeneity

From Micro to Macro

Summary

Equilibrium: A Consistency Concept

Equilibrium = mutual consistency of actions and beliefs.

Not stability. An equilibrium need not be dynamically stable; nothing in the definition implies convergence.

Not efficiency. Competitive equilibria are efficient only under strong assumptions (no distortions, no externalities, complete markets).

Not uniqueness. Multiple equilibria arise naturally in dynamic and strategic environments.

Not perfect foresight. Equilibria can be stochastic, driven by learning, or self-fulfilling. What is required: agents optimize given $\mathcal{I}_t$, and beliefs are consistent with the perceived law of motion.

Implication: In a recursive equilibrium, the perceived law of motion G is itself an equilibrium object—not an exogenous input.

Why Use Equilibrium at All?

Critique: "The real economy is never in equilibrium."

Response:

- Equilibrium is not a claim about the world—it is a **prediction** of a model given its assumptions.
- It provides a mapping: environment $\longrightarrow$ outcomes (allocations, prices, welfare).
- It allows **counterfactual policy evaluation**: change primitives, recompute, compare.

Macro is comparative equilibrium analysis.

We change the environment $\Rightarrow$ compute a new equilibrium $\Rightarrow$ compare outcomes.

Today: We do exactly this—write down the neoclassical growth model, then ask what changes when we add heterogeneity and distortions.

Roadmap

Understanding Equilibrium

The Neoclassical Growth Model

Adding Heterogeneity

From Micro to Macro

Summary

The Neoclassical Growth Model: Motivation

The neoclassical growth model is the canonical benchmark for dynamic general equilibrium analysis.

Kaldor's Stylized Facts (1957):

1. Output per capita Y/L grows at a roughly constant rate
2. Capital-output ratio K/Y is approximately constant
3. Capital-labor ratio K/L grows at the rate of output growth
4. Labor share wL/Y is approximately constant
5. Real wages grow with productivity
6. Real interest rate is stationary
7. Hours per capita are roughly constant

Goal: Construct an economy whose balanced growth path is consistent with all seven facts simultaneously.

Environment: Technology

Time: Discrete, $t = 0, 1, 2, \ldots$. No uncertainty (for now).
Production. A representative firm operates

$$Y_t = F(K_t, A_t n_t),$$

where $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ is C^2, strictly concave, constant returns to scale, and satisfies Inada conditions:

$$\lim_{K \to 0} F_1 = \infty, \qquad \lim_{K \to \infty} F_1 = 0.$$

Technological progress is labor-augmenting:

$$A_{t+1} = (1+g)A_t, \qquad g > 0, \quad A_0 > 0.$$

This is not a modeling choice—it is a **requirement** for balanced growth (Uzawa, 1961).
Capital accumulation:

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad \delta \in (0,1), \quad K_0 > 0 \text{ given}.$$

Environment: Preferences

A representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t), \qquad \beta \in (0,1),$$

where $u : \mathbb{R}_{++} \times (0,1] \to \mathbb{R}$ is C^2, strictly concave, strictly increasing in both arguments, and satisfies Inada conditions.

Balanced growth compatibility (King-Plosser-Rebelo, 1988). Preferences must admit a balanced growth path with $n_t = \bar{n}$ constant and c_t growing at rate g. This requires income and substitution effects of wage growth on labor supply to cancel exactly.

Two admissible classes:

$$u(c,\ell) = \frac{\left(c^\theta \ell^{1-\theta}\right)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0,\ \sigma \neq 1, \quad \ell = 1-n \qquad \text{(Cobb-Douglas)}$$

$$u(c,n) = \log c + \chi \frac{n^{1-1/\psi}}{1-1/\psi}, \quad \chi > 0,\ \psi > 0 \qquad \text{(Log + Frisch)}$$

Sequential Competitive Equilibrium

Definition (Sequential Competitive Equilibrium)

A sequential competitive equilibrium (SCE) is a sequence of allocations $\{c_t, n_t, K_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, R_t\}_{t=0}^{\infty}$ such that:

1. Household. Given $\{w_t, R_t\}_{t=0}^{\infty}$ and $k_0 = K_0$:

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

$$\text{s.t.} \quad c_t + k_{t+1} = w_t n_t + R_t k_t, \quad c_t \geq 0, \quad n_t \in [0,1], \quad \forall\, t \geq 0,$$

and the no-Ponzi condition (ruling out unbounded borrowing): $\lim_{T\to\infty} k_{T+1} \prod_{s=1}^{T} R_s^{-1} \geq 0$.

2. Firm. In each t, competitive profit maximization yields:

$$w_t = F_2(K_t, A_t n_t) \cdot A_t, \qquad R_t = F_1(K_t, A_t n_t) + 1 - \delta.$$

3. Market clearing. $c_t + K_{t+1} - (1 - \delta) K_t = F(K_t, A_t n_t)$.

Necessary and Sufficient Conditions for the Household

First-order conditions. An interior solution $\{c_t, n_t, k_{t+1}\}$ satisfies:

Euler equation (intertemporal):

$$u_1(c_t, 1 - n_t) = \beta\, R_{t+1}\, u_1(c_{t+1}, 1 - n_{t+1})$$

Labor-leisure (intratemporal):

$$\frac{u_2(c_t, 1 - n_t)}{u_1(c_t, 1 - n_t)} = w_t$$

Transversality:

$$\lim_{T \to \infty} \beta^T\, u_1(c_T, 1 - n_T)\, k_{T+1} = 0$$

Sufficiency. Under strict concavity of u and Inada conditions, the Euler equation, intratemporal condition, budget constraint, and transversality condition are jointly **necessary and sufficient** for the household's optimum.

Welfare Theorems

Theorem (First Welfare Theorem). Let (x^*, y^*, ν^*) be a competitive equilibrium with commodity space $\mathcal{L}$, consumption set X, and production set Y. Suppose local nonsatiation holds. Then (x^*, y^*) is Pareto efficient.

Theorem (Second Welfare Theorem). Suppose X is convex, Y is convex with nonempty interior, and u is continuous and quasi-concave. Then for any Pareto efficient allocation (x^*, y^*), there exists a continuous linear functional ν such that (x^*, y^*, ν) is a quasi-equilibrium.

Implication for the growth model: Under our assumptions (complete markets, no externalities, convex technology, strictly concave u), the SPP has a unique solution, so:

$$\text{CE allocation} \;=\; \text{Planner's allocation.}$$

Prices can be recovered from the planner's first-order conditions.

Failures: Externalities, distortionary taxes, incomplete markets, non-convexities.

From Sequences to Recursive Formulation

The welfare theorems give us the **sequence problem** of the social planner:

$$\max_{\{c_t, n_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

$$\text{s.t.} \quad c_t + K_{t+1} = F(K_t, n_t) + (1-\delta)K_t, \qquad K_0 \text{ given.}$$

Note: We suppress A_t throughout—assume stationarized or $g = 0$ for now.

Problem: This involves optimizing over an **infinite-dimensional** object $\{c_t, K_{t+1}\}_{t=0}^{\infty}$.

Key observation: The problem has **recursive structure**. At any date t, the remaining problem from t onward has the same form as the original, with K_t as the only payoff-relevant state variable.

Bellman's Principle of Optimality: An optimal plan has the property that, whatever the initial state and initial decision, the remaining decisions must constitute an optimal plan with regard to the state resulting from the first decision.

Deriving the Bellman Equation

Claim. If $\{c_t^*, K_{t+1}^*\}_{t=0}^{\infty}$ solves the sequence problem and we define

$$V(K) := \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^*, 1 - n_{t+s}^*) \quad \text{evaluated along the optimum from } K_t = K,$$

then V satisfies the **functional equation**:

$$\boxed{V(K) = \max_{c,n,K' \geq 0} \{u(c, 1-n) + \beta V(K')\}}$$

$$\text{s.t.} \quad c + K' = F(K, n) + (1-\delta)K.$$

Proof sketch. Write the sequence objective as:

$$u(c_0, 1-n_0) + \beta \underbrace{\sum_{s=0}^{\infty} \beta^s u(c_{1+s}, 1-n_{1+s})}_{\text{value from } K_1 \text{ onward}} = u(c_0, 1-n_0) + \beta\, V(K_1).$$

Planner's Optimality Conditions (Recursive)

The Bellman equation

$$V(K) = \max_{c,n,K'} \{u(c, 1-n) + \beta V(K')\} \quad \text{s.t.} \quad c + K' = F(K,n) + (1-\delta)K$$

has first-order conditions (substituting the constraint):

Euler equation:

$$u_1 = \beta V'(K') \qquad \Longrightarrow \qquad u_1(c, 1-n) = \beta \left[F_1(K', n') + 1 - \delta\right] u_1(c', 1-n')$$

where the second form uses the **Envelope Theorem**: $V'(K) = \left[F_1(K,n) + 1 - \delta\right] u_1(c, 1-n)$.

Intratemporal:

$$u_2(c, 1-n) = u_1(c, 1-n) \cdot F_2(K, n)$$

Verification: These are **identical** to the SCE household conditions with $R_{t+1} = F_1(K_{t+1}, n_{t+1}) + 1 - \delta$ and $w_t = F_2(K_t, n_t)$. The welfare theorems guarantee this: planner and decentralized economy deliver the same allocation.

Stationarization

With $A_{t+1} = (1+g)A_t$, define per-efficiency-unit variables:

$$\tilde{k}_t = \frac{K_t}{A_t}, \qquad \tilde{c}_t = \frac{c_t}{A_t}.$$

Resource constraint. Using CRS, $F(K, An) = A \cdot F(\tilde{k}, n)$:

$$\tilde{c}_t + (1+g)\tilde{k}_{t+1} = F(\tilde{k}_t, n_t) + (1-\delta)\tilde{k}_t.$$

Euler equation. With Cobb-Douglas preferences $u = \frac{(c^\theta \ell^{1-\theta})^{1-\sigma}}{1-\sigma}$, we have $u_1 \propto c^{\theta(1-\sigma)-1}$, so:

$$\frac{u_1(\tilde{c}_{t+1}A_{t+1},\, \ell_{t+1})}{u_1(\tilde{c}_t A_t,\, \ell_t)} = \left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{\theta(1-\sigma)-1} (1+g)^{\theta(1-\sigma)-1}.$$

The stationary Euler equation becomes (where $\tilde{R}_{t+1} = F_1(\tilde{k}_{t+1}, n_{t+1}) + 1 - \delta$):

$$1 = \beta(1+g)^{\theta(1-\sigma)-1} \cdot \tilde{R}_{t+1} \cdot \left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{\theta(1-\sigma)-1}$$

Exercise: Verify Balanced Growth

Given the stationary resource constraint and Euler equation derived above, with $F(k, n) = k^{\alpha} n^{1-\alpha}$ and Cobb-Douglas preferences:

Tasks:

1. Write the planner's Bellman equation in stationary variables $V(\tilde{k})$.
2. Derive the stationary Euler equation and intratemporal condition.
3. State the transversality condition in stationary variables.
4. At the steady state $(\tilde{k}^*, \tilde{c}^*, n^*)$, verify:
   - $\rightarrow$ Y_t/L grows at rate g (Fact 1)
   - $\rightarrow$ $K_t/Y_t = \tilde{k}^*/F(\tilde{k}^*, n^*)$ is constant (Fact 2)
   - $\rightarrow$ $w_t = A_t F_2(\tilde{k}^*, n^*)$ grows at rate g (Fact 5)
   - $\rightarrow$ $R^* = F_1(\tilde{k}^*, n^*) + 1 - \delta$ is constant (Fact 6)
   - $\rightarrow$ $n^* = \bar{n}$ is constant (Fact 7)

Exercise: When Does the Recursive Formulation Break?

Setup. Consider three variants of the growth model.

(a) Habit formation. Period utility is $u(c_t, c_{t-1})$, increasing in c_t and decreasing in c_{t-1}.

1. Write the planner's sequence problem. What is the **minimal state vector** for a recursive formulation?
2. Write the Bellman equation. Is V still a function of K alone?
3. Is the CE allocation Pareto efficient? Can you use the planner's Bellman to find it?

(b) "Catching up with the Joneses." Period utility is $u(c_t, C_t)$, where C_t is aggregate consumption—an externality.

1. Does the Principle of Optimality apply to the **household's** problem? To the **planner's**?
2. Show the CE is <u>not</u> Pareto efficient. What wedge appears in the Euler equation?
3. Define an RCE for this economy. What additional aggregate state must the household track?

Exercise: When Does the Recursive Formulation Break? (cont.)

(c) Capital income tax. The government taxes capital income at rate $\tau > 0$ and rebates revenue as lump-sum transfers $T_t = \tau\, r_t\, K_t$.

1. Write the household's sequential problem. Derive the Euler equation and show the tax wedge:

$$u_1(c_t, 1 - n_t) = \beta\big[1 + (1-\tau) r_{t+1}\big]\, u_1(c_{t+1}, 1 - n_{t+1}).$$

2. Write the planner's Euler equation (no tax). Conclude that CE $\neq$ planner when $\tau > 0$.
3. Formulate the household's Bellman equation. Verify that the Bellman operator

$$(Tv)(K, a) = \max_{c, a'} \{u(c) + \beta\, v(G(K), a')\}$$

is still a contraction on bounded continuous functions. Why does the contraction argument survive even though welfare theorems fail?

Takeaway: The Principle of Optimality and the contraction mapping theorem are properties of **optimization problems**, not of equilibria. They apply to both the planner and the household. The welfare theorems govern whether the planner's problem delivers the equilibrium allocation. When it doesn't, we need RCE—but the recursive machinery still works.

Why the Planner's Shortcut Is Not Enough

The welfare theorems let us solve the planner's problem and recover the CE.

This strategy fails when:

- Distortionary taxes $\Rightarrow$ wedge between planner's and household's FOCs
- Externalities $\Rightarrow$ planner internalizes effects that households ignore
- Incomplete markets $\Rightarrow$ constrained-inefficient equilibria (Greenwald-Stiglitz, 1986)
- Non-convexities $\Rightarrow$ SWT does not apply

In all these cases:

- The CE allocation $\neq$ planner's allocation
- We cannot use the planner's Bellman equation to find the equilibrium
- But we still want recursive methods

Solution: Write the **household's** problem recursively, impose equilibrium as a fixed point in function space. This is the **Recursive Competitive Equilibrium** (RCE).

Recursive Competitive Equilibrium: Setup

Key problem: The household's sequential problem requires knowing the entire future path of prices $\{w_t, R_t\}_{t=0}^{\infty}$.

Recursive resolution: If prices are functions of a **finite-dimensional state**, the household only needs to track that state and a perceived law of motion.

Let aggregate capital K be the state. Factor prices become functions:

$$w(K) = F_2(K, 1), \qquad R(K) = F_1(K, 1) + 1 - \delta.$$

The household **perceives** that aggregate capital evolves as $K' = G(K)$ and solves:

$$V(K, a) = \max_{c, a'} \{u(c) + \beta V(K', a')\}$$

$$\text{s.t.} \quad c + a' = w(K) + R(K)\, a, \qquad K' = G(K).$$

Note: The household treats G as given. Equilibrium requires that G is **consistent** with the household's own choices.

Recursive Competitive Equilibrium: Definition

Definition (Recursive Competitive Equilibrium)

An RCE is a set of functions $\{V, g, w, R, G\}$ such that:

1. **Household optimality.** Given $w(\cdot)$, $R(\cdot)$, and $G(\cdot)$, the value function $V(K, a)$ solves the household's Bellman equation and $g(K, a)$ is the associated policy function.
2. **Firm optimality.** Factor prices satisfy:

$$w(K) = F_2(K, 1), \qquad R(K) = F_1(K, 1) + 1 - \delta.$$

3. **Consistency (fixed point).** When $a = K$:

$$g(K, K) = G(K).$$

Recursive Competitive Equilibrium: Intepretation

Interpretation: Equilibrium is a fixed point in **function space**. The individual policy g, evaluated at $a = K$, must reproduce the aggregate law of motion G that the household took as given.

Key advantage: This definition does not invoke welfare theorems. It applies with externalities, taxes, incomplete markets—anywhere the planner's shortcut fails.

Note: Market clearing is not stated separately—it is implied by the consistency condition and Walras' law.

Roadmap

Understanding Equilibrium

The Neoclassical Growth Model

Adding Heterogeneity

From Micro to Macro

Summary

Heterogeneity in Wealth: Setup

Environment. Two types $i \in \{R, P\}$ of measure μ and $1-\mu$. Agents share preferences u and discount factor β. They differ **only** in initial wealth a_0^i. No uncertainty. Inelastic labor supply $N = 1$.

Individual problem. An agent of type i with wealth a solves:

$$V(K^R, K^P, a) = \max_{c,a'} \left\{ u(c) + \beta V(K^{R\prime}, K^{P\prime}, a') \right\}$$

subject to:

$$c + a' = w\big(\underbrace{\mu K^R + (1-\mu)K^P}_{=: K}\big) + a \cdot R(K), \qquad K^{i\prime} = G^i(K^R, K^P)$$

Key observation: K is a sufficient statistic for factor prices, but **not** for the laws of motion G^i. Generically, decision rules are nonlinear in $a \Rightarrow$ cannot collapse (K^R, K^P) into K.

Steady-State Indeterminacy of the Wealth Distribution

Step 1: Euler equations. In steady state ($a^{i\prime} = a^i$, $c^{i\prime} = c^i$), the Euler equation for each type reduces to:

$$u'(c^{i*}) = \beta R\, u'(c^{i*}) \quad \Longrightarrow \quad \beta R = 1, \qquad R = F_K(K^*, 1)$$

This is the **same** equation for both types (common β, common R) $\Rightarrow$ **one equation**, pinning down K^*.

Step 2: Budget constraints. With $a^{i\prime} = a^i$:

$$c^{i*} = w(K^*) + a^{i*}\big(R(K^*) - 1\big) \qquad \text{for } i = R, P$$

Step 3: Equation count.

Equations: $\beta R = 1$ [1] + budget^R, budget^P [2] $= \mathbf{3}$

Unknowns: $a^{R*},\ a^{P*},\ c^{R*},\ c^{P*} \quad = \mathbf{4}$

Result: One degree of freedom. Any split (a^{R*}, a^{P*}) satisfying $\mu a^{R*} + (1-\mu)a^{P*} = K^*$ is a steady state. The model is **silent about the distribution of wealth**.

Gorman Aggregation

Theorem (Gorman Aggregation). Suppose all agents share common preferences u and discount factor β, and the individual savings policy function takes the form:

$$g(K, a) = \bar{\mu}(K) + \lambda(K)\, a$$

where $\lambda(K)$ is a **common** marginal propensity to save. Then aggregate capital satisfies:

$$K' = \sum_i \mu_i\, g(K, a^i) = \sum_i \mu_i \left[\bar{\mu}(K) + \lambda(K)\, a^i\right] = \bar{\mu}(K) + \lambda(K)\, K$$

Result: K' depends only on K, not on the distribution $\{a^i\}$. A representative agent with wealth K makes the same aggregate choice.

Sufficient conditions: Homothetic preferences + common β + complete markets

Generalization: With n types differing only in initial wealth, there is a continuum of dimension $n - 1$ of steady-state wealth distributions—all consistent with the same K^*.

When Gorman Aggregation Fails

Aggregation requires $g(K, a) = \bar{\mu}(K) + \lambda(K)\, a$ with λ **common** across agents.

This fails when:

- Heterogeneous discount factors $\beta^i \Rightarrow$ different Euler equations $\Rightarrow \lambda^i \neq \lambda^j$
- Heterogeneous risk aversion σ^i
- Incomplete markets / borrowing constraints $\Rightarrow$ policy functions are **kinked** at the constraint, destroying linearity

Consequence: When aggregation fails, the wealth distribution $\{a^i\}$ becomes a **state variable** of the economy. Individual policies no longer collapse into a representative agent.

This is why we need Aiyagari (1994) and Huggett (1993).

These models break Gorman by introducing idiosyncratic risk + borrowing constraints $\Rightarrow$ nonlinear savings $\Rightarrow$ the distribution is an equilibrium object.

Roadmap

From Aggregation to Identification

Theoretical question: When does heterogeneity matter for aggregate outcomes?

- Under strong aggregation conditions (e.g. Gorman), distributions wash out.
- Otherwise, equilibrium aggregates depend on the full distribution.

Empirical question: If heterogeneity affects aggregates, can cross-sectional variation identify aggregate policy effects?

Answer: No.
Cross-sectional regressions identify partial equilibrium slopes, not the general equilibrium intercept shift.

Conclusion: Recovering aggregate effects requires a structural general equilibrium model.

The Missing Intercept Problem (Benjamin Moll)

Question: Can we learn about aggregate effects from cross-sectional variation?

Context: Fiscal stimulus $\Rightarrow$ output, consumption?

- x_{it}: government spending in region i at time t
- y_{it}: GDP in region i at time t
- $X_t = \frac{1}{N}\sum_i x_{it}$: aggregate government spending
- $Y_t = \frac{1}{N}\sum_i y_{it}$: aggregate GDP

Question we want to answer: What's the effect of X_t on Y_t?

Problem: Could regress Y_t on X_t (VAR), but often don't believe identification off time-series $\Rightarrow$ use cross-sectional variation instead.

The Missing Intercept: Setup

Suppose the true model is:

$$y_{it} = \alpha + \beta x_{it} + \gamma X_t + \varepsilon_{it}$$

Typical strategy: Estimate with time fixed effects

$$y_{it} = \tilde{\alpha}_t + \beta x_{it} + \varepsilon_{it}, \qquad \tilde{\alpha}_t := \alpha + \gamma X_t$$

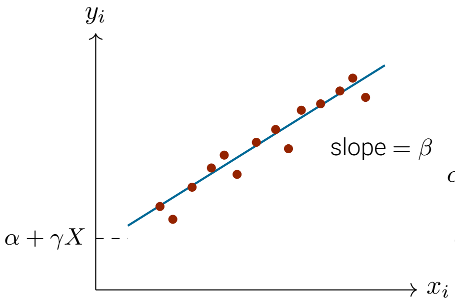
Problem: No cross-sectional variation in aggregate $X_t \Rightarrow$ absorbed into intercept!

Naive exercise concludes: aggregate relationship is $\Delta Y_t = \beta \times \Delta X_t$.

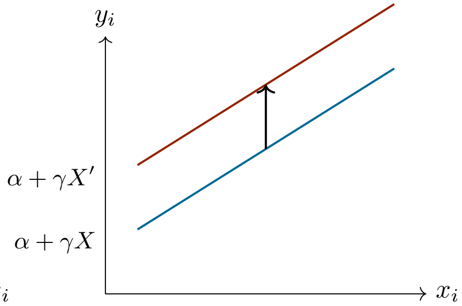
This is wrong! The true aggregate elasticity $\neq \beta$.

Logic: Cross-sectional variation identifies the **slope** but not the **intercept**. But the intercept is what we really care about!

The Missing Intercept: Graphical Intuition



Cross-section identifies slope
but not intercept



Shifts in X shift entire line
(the "missing intercept")

Key insight: Cross-sectional variation only identifies **relative effects**, but we care about **absolute effects**.

The Missing Intercept: Examples

Examples of the missing intercept problem:

1. **China shock:** x = import competition, y = employment (Autor-Dorn-Hanson)
2. **Household balance sheets:** x = housing net worth, y = consumption (Mian-Sufi)
3. **Bank lending:** x = bank lending, y = firm production (Chodorow-Reich)
4. **Unemployment benefits:** x = UI benefits, y = unemployment
5. **Stock market wealth effect:** x = stock wealth, y = consumption
6. **Monetary policy:** x = housing equity, y = refinancing (Beraja et al.)
7. **Consumer bankruptcy:** x = debt forgiveness, y = employment

Example: Autor-Dorn-Hanson (2013) claim "import competition explains one-quarter of aggregate decline in US manufacturing employment" by scaling regression coefficient—but this requires **very strong assumptions**!

Strategies for Recovering the Missing Intercept

In short: Need more structure... i.e., a “model” (in the general sense).

Candidate strategies:

1. Using full-blown models to convert regional estimates into partial and general equilibrium effects
   - → Nakamura-Steinsson, Guren-McKay-Nakamura-Steinsson
   - → Chodorow-Reich-Nenov-Simsek, Auclert-Dobbie-Goldsmith-Pinkham
2. Using a bit of structure + VAR estimates
   - → Wolf, Beraja-Hurst-Ospina

My guess: No general solution—expect solution to depend on particular application.

Conclusion: Whatever you do, please don’t just scale up micro regression coefficients!

Reference: Ben Moll’s reading list: `http://benjaminmoll.com/micro_to_macro/`

Roadmap

Understanding Equilibrium

The Neoclassical Growth Model

Adding Heterogeneity

From Micro to Macro

Summary

Key Takeaways

1. A **model** is an artificial economy; not complete without an **equilibrium concept**
2. **Equilibrium** = mutual consistency of actions and expectations
   $\rightarrow$ Does not imply stability, optimality, uniqueness, or perfect information
3. **Neoclassical Growth Model** matches Kaldor's stylized facts
   $\rightarrow$ Requires labor-augmenting technology, CRS, and balanced growth preferences
4. **Welfare theorems** connect competitive equilibria and Pareto optima
   $\rightarrow$ Fail with externalities, incomplete markets, distortions
5. **RCE** works even when welfare theorems fail
6. **Missing intercept problem:** Cross-sectional variation identifies relative effects, not aggregate effects $\Rightarrow$ need structural models

Readings and Next Time

Readings for today's lecture: *AKMM*, Chapters 1-5.

Lectures 2 & 3: Dynamic programming—theory and modern computational tools.

- Contraction mappings, VFI, policy iteration, endogenous grid methods
- Reading: Sargent & Stachurski, *Dynamic Programming*, Chapters 1–5