A Macroeconomic Perspective on Taxing Multinational Enterprises*

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Abstract

We develop a framework to study the macroeconomic implications of taxing multinational enterprises (MNEs) that shift profits to subsidiaries in low-tax jurisdictions by transferring ownership of nonrival intangible capital. We first prove analytically that profit shifting increases intangible investment, leading to higher profits and output at the MNE level. We then calibrate our model so that it reproduces salient country-level facts about production, trade, FDI, and, most importantly, profit shifting. We use our calibrated model to evaluate the consequences of two proposals by the OECD and G20 governments to reduce profit shifting by MNEs: allocating the rights to tax some of an MNE's profits to the countries in which it sells its products; and a 15% minimum global corporate income tax. We show that these policies would reduce profit shifting by more than two-thirds, but would also reduce output in all regions of the global economy. This highlights a key tension for policymakers: profit shifting erodes high-tax countries' tax bases, but it also boosts economic activity worldwide.

Keywords: Multinational enterprise; transfer pricing; profit shifting; base erosion; intangible capital; corporate tax.

JEL Codes: E6, F23, H25, H27

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1 Introduction

Multinational enterprises (MNEs) shift large portions of their profits to foreign tax havens, costing governments in their home countries hundreds of billions of dollars per year in tax revenue. In October 2021, 137 countries signed onto a policy designed by the OECD and G20 governments to reduce profit shifting, making it the largest international tax reform in history. Using a new theory that links profit shifting to MNEs' production decisions, we show that profit shifting has positive effects on the real economy: it increases investment in intangible capital, boosting output in every country around the world in which MNEs operate. We embed our theory in a general-equilibrium framework, discipline it using data on the scale of profit shifting under the current international tax regime, and measure the macroeconomic consequences of the OECD/G20 proposal. While this reform would substantially reduce profit shifting and increase tax revenues in high-tax countries, it would also cause output to fall in high- and low-tax countries alike.

Base erosion and profit shifting (BEPS) refers to MNEs' use of tax planning strategies to exploit gaps and mismatches in tax rules to artificially shift profits to low- or no-tax countries where they conduct little or no economic activity, or to erode tax bases through deductible payments such as interest or royalties. The scale of profit shifting is striking. For example, Tørsløv, Wier, and Zucman (2022) estimate that 36 percent of worldwide multinational profits are shifted to tax havens, while Guvenen, Mataloni, Rassier, and Ruhl (2022) find that 38 percent of foreign income reported by U.S. MNEs is actually generated at home in the United States. The implications for public finances are equally striking: Clausing (2020a) estimates that about a third of U.S. corporate income taxes are lost to profit shifting, which is equivalent to more than $100 billion per year. According to the OECD, profit shifting reduces global corporate income tax revenues by as much as 10 percent per year, or $240 billion (Johansson et al., 2017). Consequently, addressing this issue is a top priority for policymakers in high-tax countries, where many of the biggest MNEs are based. The OECD/G20 Inclusive Framework on BEPS outlines two major policy changes, or "pillars".[1] The first pillar is revenue-based profit allocation, which allocates the rights to tax some of an MNE's profits to the countries in which it operates in proportion to these countries' shares of the MNE's global sales. The second is a global minimum corporate income tax, which would require that all corporate income, regardless of where it is booked, be effectively taxed at no lower than 15 percent.

In order to study the macroeconomic effects of profit shifting and the two-pillar OECD policy designed to address it, we develop a model in which MNEs shift profits by transfer-

[1] The press statement and a description of these pillars can be found here. We provide an executive summary of the current international tax regime and the OECD/GS20 Framework in Appendix A.

ring intangible capital property rights to foreign subsidiaries in low-tax countries.[2] As in McGrattan and Prescott (2010), intangible capital is nonrival: MNEs produce it by doing research and development at home, but use it to produce simultaneously in all of their foreign subsidiaries around the world. According to transfer pricing rules, these subsidiaries pay licensing fees to use this capital. Normally, these fees are paid to the domestic parent corporation, but the rights to this capital can be transferred—at a cost—to subsidiaries in low-tax regions (a.k.a. tax havens). The income that accrues to the MNE's intangible capital is now taxed at a lower rate, increasing the after-tax return on intangible investment. This increases the MNE's optimal level of research and development, which ultimately leads the MNE to produce more output both at home and abroad.

We use our model to make two substantive contributions, one theoretical and one quantitative. In the theoretical part of the paper, we characterize in analytic terms the impact of profit shifting on MNEs' production decisions and profitability. We prove that profit shifting increases intangible investment, leading to higher output but lower reported profits in the MNE's home country. This result clearly reveals the trade-off that profit shifting presents to policymakers: although it artificially redistributes MNEs' income to foreign tax havens, it also increases the amount of income that they actually generate. Moreover, the size of this effect is an increasing function of the difference between the corporate tax rate in the MNE's home country and the tax haven's tax rate; increasing the tax rate in the tax haven thus reduces intangible investment at home. This has direct implications for the second pillar of the OECD/G20 plan: a global minimum tax rate will reduce intangible investment, and the higher the minimum tax rate, the larger the reduction. Further, we prove that sales-based profit reallocation, the first pillar of this plan, will also have adverse effects on real economic activities.

In the quantitative part of the paper, we embed our theory of profit shifting into a general-equilibrium environment to measure the macroeconomic effects of the OECD/G20 BEPS framework. Our quantitative model features five regions that differ in population, productivity, and corporate tax rates. We split the countries identified as tax havens by Tørsløv et al. (2022) into two regions: a productive low-tax region that includes Ireland, Switzerland, and other countries where most of the economy is not devoted to profit shifting; and a tax haven that includes the Caribbean, the Channel Islands, and other small countries whose economies rely heavily on profit shifting. The other three regions are North America, Europe (minus countries in the low-tax region), and the rest of the world. Heterogeneous

[2]See Guvenen et al. (2022) and Delis et al. (2021) for evidence on the role of intangible capital in profit shifting.

firms pay fixed costs to establish foreign subsidiaries in other regions (i.e., become MNEs) as in Helpman, Melitz, and Yeaple (2004) and Garetto, Oldenski, and Ramondo (2019), and firms with subsidiaries in one of the first two regions can shift profits by transferring the rights to intangible capital according to our theory. We discipline our model using data on production, trade, multinational activity, and, most importantly, estimates of international profit shifting from Tørsløv et al. (2022). We find that the OECD's proposal would go a long way toward eliminating profit shifting: lost profits would fall by 77 percent in North America, 82 percent in Europe, and 90 percent in the rest of the world. However, it would also materially reduce intangible capital investment and overall macroeconomic performance across the globe: GDP would fall by 0.17 percent in North America, 0.16 percent in Europe, 0.13 percent in the low-tax productive region, and 0.14 percent in the rest of the world.

2 Related Literature

This paper draws on and contributes to several strands of literature. First, there are a number of studies that aim to measure the scope of international profit shifting by MNEs. Guvenen, Mataloni, Rassier, and Ruhl (2022) use confidential survey data from the Bureau of Economic Analysis (BEA) to estimate that 38 percent of income recorded by U.S. MNEs on their foreign direct investment should be re-attributed to U.S. GDP. Importantly, they document that profit shifting is concentrated in industries and firms with significant research and development spending, providing support for our theory that intangible assets are central to profit shifting. Tørsløv, Wier, and Zucman (2022) combine cross-country data on wages and profitability of foreign firms' local affiliates (a.k.a. foreign affiliates statistics) versus local firms. Their main finding is that 36% of multinational profits were shifted to tax havens globally in 2015. Clausing (2020a) conclude, based on several different estimates, that the U.S. tax revenue loss from profit shifting in 2017 likely exceeded $100 billion, or about a third of federal corporate tax revenues.[3] Our interpretation of these estimates is that profit shifting is a large and consequential problem at the global scale.[4] Our contribution to this strand of literature is to develop a quantitative macroeconomic framework to assess the impact of transfer pricing and profit shifting on macroeconomic aggregates and tax revenues.

[3]These findings are in line with the $4.2 trillion in offshore earnings observed in the U.S. data, $3.0 trillion of which is in known tax havens or countries with effective tax rates below 10 percent.

[4]Blouin and Robinson (2020) and Clausing (2020a) discuss the methodological challenges associated with estimating the magnitude of profit shifting. Bolwijn, Casella, and Rigo (2018) and Crivelli, De Mooij, and Keen (2015) study the impact of profit shifting on tax revenues for developing countries. See Dowd, Landefeld, and Moore (2017), Clausing (2016), and OECD (2015) for extensive reviews of the profit-shifting literature and the estimates found therein.

We exploit the empirical estimates discussed above to discipline our structural model.

We also contribute to the literature on the macroeconomic role of intangible capital. The importance of intangible capital for aggregate measurement has been highlighted by Corrado, Hulten, and Sichel (2009), McGrattan and Prescott (2010), O'Mahony, Corrado, Haskel, Iommi, Jona-Lasinio, and Mas (2018), and Koh, Santaeulàlia-Llopis, and Zheng (2020); Peters and Taylor (2017) and Ewens, Peters, and Wang (2019) demonstrate its importance for firm-level measurement. Importantly, Delis, Delis, Laeven, and Ongena (2021) establish a causal, positive relationship at the firm level between profit shifting and the share of total assets that are intangible.[5] On the modelling front, McGrattan and Prescott (2010) develop a multi-country, general equilibrium model that includes two types of intangible capital: rival, plant-specific intangible capital and nonrival technology capital that can be used in multiple locations simultaneously. They use their model to explain the differences between the investment returns of foreign subsidiaries of U.S. multinational companies and the returns of U.S. subsidiaries of foreign multinationals. We contribute to this line of research by developing a novel model of transfer pricing and profit shifting that centers around nonrival intangible capital. In our framework, firms that shift profits have more intangible capital, consistent with the data.

This paper also relates to the macro public finance literature on corporate income taxation. This strand of research is vast and dates back to seminal contributions by Harberger (1962) and Auerbach (1983), among others. More recently, Barro and Furman (2018) assess the macroeconomic consequences of the Tax Cuts and Jobs Act of 2017 (TCJA). Kaymak and Schott (2018) argue that falling corporate income taxes across the world are the main driver behind the decline of the labor share of income. Kaymak and Schott (2019) argue that loss-offset provisions in the corporate income tax code give rise to capital misallocation and assess the associated aggregate output losses. Bhandari and McGrattan (2020) quantify the impact of reducing corporate income taxes in a model where firms choose the legal form of business organization endogenously. In spite of the large number of studies in this literature, little attention has been paid to the macroeconomic effects of international profit shifting and its impact on intangible investment. This paper aims to fill this gap.

Furthermore, our work contributes to the the international economics literature on multinational firms. See Antrás and Yeaple (2014) for a review of this extensive line of research. We extend the Helpman et al. (2004) framework by incorporating both nonrival, intangible

[5]Specifically, they estimate that a one-standard-deviation increase in the ratio of intangible assets to total assets increases profit shifting by approximately 3 percent. The intangible ratio is the characteristic with the largest impact on profit shifting in their analysis.

capital and profit shifting into the multinational firm's decision problem.[6] Both features are central for understanding the growing impact of multinational firms on the global economy.

Last, our paper relates to a large, influential literature on international tax competition.[7] The growing importance of profit shifting has led to a rapid development of this literature in recent years; see Keen and Konrad (2013) for a review. Among the most important papers in this line of research are Haufler and Schjelderup (2000), Mintz and Smart (2004), Hong and Smart (2010), and Slemrod and Wilson (2009). In this paper, we do not follow the game-theoretic approach, but applying this approach to our theory of profit shifting would represent a substantial contribution to this literature; we leave this for future research.[8]

3 Theory of Profit Shifting and Intangible Investment

In order to study the real effects of international profit shifting and the OECD/G20 policy framework designed to address this phenomenon, we develop a theory that links profit shifting to intangible investment. According to our theory, MNEs shift profits by transferring the rights to nonrival intangible capital to subsidiaries in low-tax jurisdictions. This increases the after-tax return on intangible capital, which leads MNEs to increase their investment in this capital in equilibrium and ultimately produce more output. Thus, profit shifting presents policymakers in high-tax countries with a trade-off: it reduces corporate income tax revenues, but also increases overall economic activity.

3.1 Environment

Consider an MNE that operates subsidiaries in I regions. Each region $k = 1, \ldots, I$ is characterized by population N_k, total factor productivity A_k, and corporate tax rate $\tau_k \in [0, 1]$. The MNE's home region is denoted by i. Without loss of generality, we normalize the entire population across regions to unity, i.e. $\sum_{k=1}^{I} N_k = 1$. We refer to the region with the lowest tax rate, which we denote by i^*, as the tax haven, i.e., $\tau_{i^*} = \min\{\tau_1, ..., \tau_I\}$.

In each region, the MNE has access to a production technology F_k in that transforms

[6]Our model also shares some ingredients with work by Melitz (2003), Chaney (2008), Garetto, Oldenski, and Ramondo (2019), and McGrattan and Waddle (2020).

[7]The notion of international tax competition originates from a theoretical literature on capital tax competition across jurisdictions, which has roots back to Tiebout (1956) but took shape with the seminal papers of Zodrow and Mieszkowski (1986) and Wildasin (1988).

[8]In a separate paper, we incorporate our theory of profit shifting into the standard tax competition framework. See Dyrda, Hong, and Steinberg (2022).

labor ℓ_k and intangible capital z into a final good:

$$F_k(z,\ell_k) = A_k (N_k z)^{\phi} \ell_k^{\gamma}. \tag{1}$$

As in McGrattan and Prescott (2010), intangible capital is nonrival: it is purchased in the headquarters region i at the local price p_i, but it can be used in all I locations simultaneously. Its productivity is determined by the local population N_k, which proxies for the number of production locations in a given region where the intangible capital can be deployed. Labor is rented in a competitive market at wage rate w_k. We assume decreasing returns to scale, i.e., $\phi + \gamma < 1$.[9]

As a starting point, we begin by defining the MNE's profits in the standard setup (e.g., as in McGrattan and Prescott, 2010) in which foreign subsidiaries use intangible capital free of charge:

$$\pi_i = p_i \left(A_i (N_i z)^{\phi} \ell_i^{\gamma} \right) - w_i \ell_i - p_i z \tag{2}$$

$$\pi_k = p_k \left(A_k (N_k z)^{\phi} \ell_k^{\gamma} \right) - w_k \ell_k, \quad \forall k \neq i. \tag{3}$$

We refer to this as the *free transfer* (FT) scenario and denote the allocation of intangible capital in this case by z^{FT}. Our methodological innovation is to add two new ingredients to this setup: transfer pricing and profit shifting, which we do one at a time.

In the *transfer pricing* (TP) scenario, the parent division retains legal ownership of the MNE's stock of intangible capital and licenses the right to use this capital to its foreign affiliates. The accounting profits in each of the MNE's divisions in this scenario are

$$\pi_i^{TP} = \pi_i + \sum_{k \neq i} \vartheta_k(z)\, z, \tag{4}$$

$$\pi_k^{TP} = \pi_k - \vartheta_k(z)\, z \quad \forall k \neq i. \tag{5}$$

According to the arm's length principle, the licensing fees, ϑ_k, are set to the affiliates' marginal revenue products of intangible capital,

$$\vartheta_k(z) \equiv \phi p_k N_k \left(A_k (N_k z)^{\phi - 1} \ell_k^{\gamma} \right). \tag{6}$$

We denote the allocation of intangible capital in this case by z^{TP}. In this section, we assume

[9] In our quantitative model we assume constant returns to scale and monopolistic competition. In this partial equilibrium setting, the two approaches are isomorphic. We choose a decreasing returns approach here for its analytical simplicity.

that the MNE takes $\vartheta_k(z)$ as given according to the spirit of the arm's length principle; it does not internalize the effect of its choice of z on $\vartheta_k(z)$. Mathematically speaking, the MNE does not take the derivative of $\vartheta_k(z)$ when taking the first-order condition of its profit function with respect to z. This keeps our key equations relatively simple, which allows us to highlight the important economic forces at work behind our results. In Appendix E.1.1, we show that all of our analytical results hold when the MNE does internalize the effect of its choice of z on $\vartheta_k(z)$, and we allow for this effect in our quantitative analysis as well.

In the *profit shifting* (PS) scenario, the MNE's headquarter sells a fraction λ of its intangible capital to its affiliate in the tax haven, which then licenses the rights to use this capital to the parent division and the other non-haven foreign affiliates. We assume that the tax-haven affiliate buys intangible capital from the headquarters at a markdown $\varphi \leq 1$ below the competitive price, which is equal to the sum total of the licensing fees that this capital can generate, i.e., the sum of the marginal revenue products across all of the regions in which the MNE operates. Manipulating transfer prices in this way is assumed to be costly, as the multinational needs to modify its books, and possibly its real trade and investment patterns, to be able to justify the distorted transfer prices to the tax authorities. We impose the following assumption on the cost function $\mathcal{C}(\lambda)$.

Assumption 1 *Let* $\mathcal{C}(\lambda) \equiv \lambda + (1-\lambda)\log(1-\lambda)$*, implying* $\mathcal{C}'(\lambda) = -\log(1-\lambda)$*,* $\mathcal{C}(0) = 0$*,* $\mathcal{C}(1) = 1$*, and* $\lambda \in [0,1)$*.*

It is important to note that $C(\lambda)$ captures direct costs of profit shifting (e.g. increased spending on lawyers, accountants, and transfer pricing consultants), but also, in a reduced-form way, the increased risk of penalization by the government (see, e.g., Allingham and Sandmo, 1972; Rotberg and Steinberg, 2022).

Pre-tax profits in the *profit shifting* scenario are thus:

$$\pi_i^{PS} = \pi_i + z\left[\varphi\lambda\sum_k \vartheta_k(z) - \lambda\vartheta_i(z) + (1-\lambda)\sum_{k\neq i}\vartheta_k(z) - \sum_k \vartheta_k(z)\,\mathcal{C}(\lambda)\right], \tag{7}$$

$$\pi_{i^*}^{PS} = \pi_{i^*} + z\left[\lambda\sum_{k\neq i^*}\vartheta_k(z) - (1-\lambda)\,\vartheta_{i^*}(z) - \varphi\lambda\sum_k \vartheta_k(z)\right], \tag{8}$$

$$\pi_k^{PS} = \pi_k - \vartheta_k(z)\,z \quad \forall k \neq i, i^*. \tag{9}$$

The first term in the square brackets in (7), $\varphi\lambda\sum_k \vartheta_k(z)$, is the revenue from selling intangible capital to the tax haven. The second term, $-\lambda\vartheta_i(z)$, denotes the licensing fee that the headquarter pays to the tax haven for the right to use the fraction λ of intangible capital that has changed ownership. The third term, $(1-\lambda)\sum_{k\neq i}\vartheta_k(z)$, represents the licensing fees that the headquarter collects from the other affiliates for the remaining intangible capital that the headquarter retains. The term $\mathcal{C}(\lambda)\sum_k \vartheta_k(z)$ captures the costs of shifting intangible capital to the tax haven. The terms in (8) have analogous interpretations. We denote the allocation of intangible capital in this scenario by z^{PS}.

Consider the problem of maximizing after-tax profits in each scenario:

$$\max_{z^s,\left\{\ell_k^s\right\}_{k=1}^I,\lambda} \sum_{k=1}^I (1-\tau_k)\,\pi_k^s \tag{10}$$

where $s \in \{FT, TP, PS\}$. Note that λ is only chosen in the profit shifting scenario. We first characterize the MNE's optimal choice of λ in this scenario, and then characterize how this choice alters the MNE's intangible investment decision. The formal proofs of these results are relegated to Appendix E.1.

3.2 Optimal profit shifting

In the *profit shifting* scenario, the MNE's optimal choice of λ is given by

$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right). \tag{11}$$

The following lemma provides a formal characterization of how this solution depends on the profit shifting technology, which is governed by the markdown φ, and the potential gain from shifting profits, which is governed by the tax haven's tax rate τ_{i^*}.

Lemma 1 *Under Assumption 1, the share λ of intangible capital sold to the tax haven is:*

1. *decreasing in φ with elasticity given by*

$$\varepsilon_\varphi^\lambda = -\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{\tau_i-\tau_{i^*}}{1-\tau_i}\right)\varphi < 0, \tag{12}$$

and is equal to zero if $\varphi = 1$;

2. *decreasing in τ_{i^*} with elasticity given by*

$$\varepsilon_{\tau_{i^*}}^\lambda = -\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{1-\varphi}{1-\tau_i}\right)\tau_{i^*} < 0. \tag{13}$$

The first part of this lemma says that the smaller the markdown below the competitive price (i.e. the larger φ is), the smaller the fraction of intangible capital that is shifted to the tax haven. In particular, if the MNE has to sell the rights to intangible capital at the competitive price with no markdowm (i.e., $\varphi = 1$), then no profit shifting takes place at all. The second part says that λ is decreasing in the tax haven's tax rate, τ_{i^*}. The elasticity of λ with respect to τ_{i^*} depends on four terms. First, the closer λ is to 1, the larger the reduction. Second, λ is more responsive to τ_{i^*} if the markdown φ is smaller. Third, the elasticity is increasing in the level of the tax rate in the headquarters, τ_i. Finally, it is proportional to τ_{i^*} itself.

3.3 The Effect of Profit Shifting on Intangible Investment

Having characterized the MNE's decision about how much intangible capital to transfer to the tax haven, we can now characterize the effect of this decision on the MNE's intangible investment choice. The optimal intangible capital allocations in the three scenarios are

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\,\Lambda_k}{(1-\tau_i)\,p_i} \right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{14}$$

$$z^{TP} = \left(\frac{\sum_k \Lambda_k}{p_i} \right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{15}$$

$$z^{PS} = z^{TP} \left(1 - \mathcal{C}(\lambda) + \frac{\lambda(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i} \right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{16}$$

where

$$\Lambda_k \equiv \phi \gamma^{\frac{\gamma}{1-\gamma}} p_k^{\frac{1}{1-\gamma}} A_k^{\frac{1}{1-\gamma}} \left(\frac{1}{w_k} \right)^{\frac{\gamma}{1-\gamma}} N_k^{\frac{\phi}{1-\gamma}}. \tag{17}$$

The following proposition summarizes the relationships between these allocations.

Proposition 1 *Under Assumption 1, the following hold:*

1. *if* $\tau_i = \max\{\tau_k\}_{k=1}^K$ *then* $z^{TP} < z^{FT}$*;*
2. $z^{PS} > z^{TP} \iff \varphi < 1$ *and* $z^{PS} = z^{TP} \iff \varphi = 1$*;*
3. z^{PS} *is decreasing in* φ*;*
4. z^{PS} *is decreasing in* τ_{i^*} *with elasticity*

$$\varepsilon_{\tau_{i^*}}^{z^{PS}} = -\left(\frac{1-\gamma}{1-\phi-\gamma} \right) \frac{1}{\left(1 + \frac{1-\mathcal{C}(\lambda)}{\lambda \mathcal{C}'(\lambda)}\right)} \left(\frac{\tau_{i^*}}{\tau_i - \tau_{i^*}} \right) < 0. \tag{18}$$

The first part of the proposition states that if the MNE's home country has the highest tax rate across all of the jurisdictions in which the MNE operates, transfer pricing reduces intangible investment, i.e., $z^{TP} < z^{FT}$. Intuitively, requiring foreign affiliates to pay licensing fees to use intangible capital reallocates intangible income to the headquarters, and if the headquarters' income is taxed at a higher rate, the MNE's global profits decline. This demonstrates that asymmetries in tax rates across jurisdictions are more distortionary when MNEs are required to account for intangible income according to the arm's length principle.

The second part of the proposition states that, relative to the transfer pricing scenario, profit shifting increases intangible investment, i.e., $z^{PS} > z^{TP}$, if and only if intangible capital can be sold to the tax haven below the competitive price, i.e., $\varphi < 1$. In this case, as can be seen in (11), $\lambda \in (0, 1)$, and we show in the Appendix that this implies the term in parentheses in (16) is strictly greater than one. Intuitively, profit shifting allows the MNE to partially undo the impact of transfer pricing. Transfer pricing forces the MNE to book foreign affiliates' intangible income at the home tax rate, while profit shifting allows the MNE to book some of this income at the tax haven's tax rate instead. In fact, if the MNE's home country has the highest tax rate, then one can show that $z^{TP} < z^{PS} < z^{FT}$.

The third and fourth parts of the proposition characterize the size of the effect described in the second part. As shown in Lemma 1, the smaller the markdown (the larger φ is), the smaller the fraction λ of intangible capital that is sold to the tax haven. This implies that the MNE's profit is decreasing in φ; the closer the transfer price is to the competitive price, the lower the incentive to purchase intangible capital. In turn, this implies that z^{PS} is decreasing in φ. Similarly, z^{PS} is decreasing in the tax haven's tax rate τ_{i^*}. As this rate increases, λ falls, and with it falls the extra gain from intangible investment relative to the transfer pricing scenario. The elasticity of this margin is negative and given by (18). It is a product of three terms: (i) technological parameters; (ii) the profit shifting cost function; and (iii) the difference between the tax rates in the tax haven and the MNE's home country. These comparative statics are also illustrated in Figure 2 in the Appendix.

These results are crucial for understanding the central economic trade-off we uncover in this paper: profit shifting erodes high-tax countries' tax bases, but also boosts economic activity by increasing MNEs' intangible investment. This trade-off has important implications for the OECD/G20 BEPS framework. Specifically, a global minimum corporate income tax—which in this simple environment acts like an increase in the tax haven's tax rate—will reduce profit shifting, but this reduction will come at the cost of lower economic performance.

3.4 The Effect of the Profit Allocation Rule

We can also use our theory of profit shifting to illustrate the impact of the first pillar of the OECD/G20 framework, which allocates the rights to tax a portion of an MNE's global profits to the regions in which it operates in proportion to these regions' shares of the MNE's overall sales. Under this rule, the tax base of a subsidiary in region k is the sum of local routine profit π_k^r, a share $(1-\theta)$ of local residual profit π_k^R, and a fraction of total global residual profit Π^R that is based on this region's share of the MNE's total global sales:

$$T_k = \pi_k^r + (1-\theta)\cdot \pi_k^R + \theta \cdot \frac{p_k y_k}{\sum_k p_k y_k} \cdot \Pi^R. \tag{19}$$

Routine profit is defined as the fraction μ of the revenues in jurisdiction k,

$$\pi_k^r = \mu p_k y_k, \tag{20}$$

and residual profit is defined as the complementary fraction,

$$\pi_k^R = \pi_k^{PS} - \pi_k^r. \tag{21}$$

Global residual profit is the sum of residual profits across regions:

$$\Pi^R = \sum_i \pi_i^R. \tag{22}$$

The two key parameters are: (i) the fraction of residual profits that are allocated across regions based on sales, θ; and (ii) the routine profitability margin, μ. Under the OECD/G20 proposal, these are set to $\theta = 0.25$ and $\mu = 0.1$, but in what follows we will analyze comparative statics with respect to their values.

Consider now the MNE's modified profit-maximization problem in the *profit shifting* scenario under the profit allocation rule:

$$\max_{z^{PS},\left\{\ell_k^{PS}\right\}_{k=1}^{I},\lambda} \sum_{k=1}^{I} \left(\pi_k^{PS} - \tau_k T_k\right). \tag{23}$$

The share of intangible capital that is sold to the tax haven is now given by

$$\hat{\lambda} = 1 - \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-((1-\theta)\tau_i + \theta\widehat{\tau})}\right). \tag{24}$$

where $\widehat{\tau}$ is the sales-weighted average tax rate across regions:

$$\widehat{\tau} \equiv \sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}. \tag{25}$$

The MNE's optimal choice of intangible capital is given by

$$\hat{z}^{PS} = \hat{z}^{TP} \left(1 - \mathcal{C}\left(\lambda\right) + \frac{\left(1-\theta\right)\lambda\left(1-\varphi\right)\left(\tau_i - \tau_{i^*}\right)}{\left(1-\left(\left(1-\theta\right)\tau_i + \theta\widehat{\tau}\right)\right)} \right)^{\frac{1-\gamma}{1-\phi-\gamma}}. \tag{26}$$

We are now ready to characterize how the profit allocation rule affects the MNE's intangible investment decision.

Proposition 2 *Under Assumption 1, the following hold:*

1. *the allocation of intangible capital under the profit allocation rule, for any $0 < \theta \leq 1$, is smaller than under the current regime, i.e. $\widehat{z}^{PS} < z^{PS}$;*

2. *$\hat{z}^{PS}$ is decreasing in θ with elasticity given by*

$$\varepsilon_{\theta}^{\hat{z}^{PS}} = \varepsilon_{\theta}^{\hat{\lambda}} \left(\frac{1-\gamma}{1-\phi-\gamma}\right) \left(\frac{\hat{\lambda}}{\mathcal{C}(\hat{\lambda})(1-\hat{\lambda})}\right) \left(\frac{1}{1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}}\right) < 0; \tag{27}$$

3. *$\hat{z}^{PS}$ is decreasing in τ_{i^*}, and if the MNE's sales in the tax haven are sufficiently small then*

$$\left|\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}}\right| < \left|\varepsilon_{\tau_{i^*}}^{z^{PS}}\right|. \tag{28}$$

The first part of the proposition states that the profit allocation rule will reduce intangible investment relative to the current regime, i.e., $\widehat{z}^{PS} < z^{PS}$. This can be seen by comparing the solution for z^{PS} in (15) with the solution for $\widehat{z}^{PS}$ in (26). The second part states that intangible investment is decreasing in the fraction of residual profits allocated based on sales, θ. The elasticity of this margin is given by (27). It is proportional to the elasticity of $\hat{\lambda}$ with respect to θ given by (91), which itself is negative as shown in the Appendix. Finally, the third part of the proposition states that intangible investment under the profit allocation rule is decreasing in the tax haven's tax rate, which is also true under the current regime. However, as with the share of intangible capital sold to the tax haven, the size of this effect is smaller under the profit allocation rule, provided that the tax haven is sufficiently small. These comparative statics are illustrated in Figure 2 in the Appendix.

These findings reveal an important interaction between the two OECD/G20 pillars and provide a deeper understanding of the trade-offs that policymakers face. On the one hand, the profit allocation rule decreases profit shifting. On the other hand, although it decreases intangible investment, it also alleviates the negative impact of the global minimum tax. As we will see, these margins play important roles in our quantitative analysis, which we take up in the next two sections of the paper.

4 Quantitative Model

In order to assess the macroeconomic implications of our theory of profit shifting, we integrate it into a general equilibrium model with heterogeneous firms in the tradition of the international economics literature. Our quantitative framework synthesizes Helpman, Melitz, and Yeaple (2004) and McGrattan and Prescott (2010). There are I "productive" regions, each populated by a representative household, a measure of heterogeneous firms, and a government. Regions, indexed by i and j, differ in population, total factor productivity, trade costs, FDI costs, and corporate income taxes. Firms in each region decide the following: where to export and where to open foreign subsidiaries; how much labor to hire in the parent division and each subsidiary; and how much intangible capital to produce in the parent division. Intangible capital is nonrival and is used simultaneously in all of a firm's divisions.

As in section 3, multinational firms (firms with foreign affiliates) use transfer pricing to allocate the costs of producing intangible capital across their foreign affiliates in proportion to the scale at which these affiliates use this capital. Affiliates license the right to use intangible capital from the division that owns this capital, and MNEs can shift profits by selling their intangible capital to affiliates in lower-tax regions. We denote the "productive" region with the lowest corporate income tax rate by LT. Additionally, there is an "unproductive" tax haven that is populated by a representative household and a government, labelled as TH, where no economic activity takes place. MNEs based in high-tax regions can transfer their intangible capital rights to either (or both) of these regions, provided that they have established affiliates there.

4.1 Households

Each region i has a representative household with preferences over consumption, C_i, and labor supply, L_i, given by

$$u\left(\frac{C_i}{N_i}, \frac{L_i}{N_i}\right) = \log\left(\frac{C_i}{N_i}\right) + \psi_i \log\left(1 - \frac{L_i}{N_i}\right). \tag{29}$$

Households choose consumption and labor supply to maximize utility subject to a budget constraint

$$P_i C_i = W_i L_i + D_i + T_i, \tag{30}$$

where W_i is the wage, D_i is the aggregate dividend payment from firms based in region i, and T_i is a transfer from the government.

Consumption is a constant-elasticity-of-substitution aggregate of products from different source countries,

$$C_i = \left[\sum_{j=1}^{J} \int_{\Omega_{ji}} q_{ji}(\omega)^{\frac{\rho-1}{\rho}} d\omega\right]^{\frac{\rho}{\rho-1}}, \tag{31}$$

where $q_{ji}(\omega)$ is the quantity of variety ω from region j, Ω_{ji} is the set of goods from j available in i (determined by firms' exporting and FDI decisions specified later), and ρ is the elasticity of substitution between varieties. The demand curve for each variety can be written as

$$p_{ji}(\omega) = P_i C_i^{\frac{1}{\rho}} q_{ji}(\omega)^{-\frac{1}{\rho}}. \tag{32}$$

The aggregate price index is

$$P_i = \left[\sum_{j=1}^{J} \int_{\Omega_{ji}} p_{ji}(\omega)^{1-\rho} d\omega\right]^{\frac{1}{1-\rho}}. \tag{33}$$

4.2 Firms

Each productive region i has a unit measure Ω_i of firms that compete monopolistically as in Melitz (2003) and Chaney (2008). Each firm is associated with a product variety ω. Firms are heterogeneous in productivity, a, which is drawn from a distribution $F_i(a)$. Firms produce their products using labor and intangible capital. Intangible capital, which we denote by z, is nonrival: it is produced in the home country but can be used to produce abroad as well, provided that a firm pays the cost of setting up a foreign affiliate in another productive region. Foreign affiliates pay licensing fees to use intangible capital according to the rules of transfer pricing. Firms can shift the profits associated with these fees to the low-tax region and/or the tax haven by transferring the rights to intangible capital to affiliates in these regions. Profit shifting is costly, however, and the more capital that is transferred, the larger the cost. Throughout this subsection, we index firms by their productivities instead of their varieties to economize on notation; all firms from a given region with the same productivity make the same decisions.

Production. A firm from region i with productivity a and intangible capital z can produce its good in any productive region j using the technology

$$y_{ij} = \sigma_{ij} A_j a \left(N_j z\right)^{\phi} \ell_j^{\gamma}. \tag{34}$$

This technology is the same as in the theory developed in section 3 with two modifications: it depends on the firm's idiosyncratic productivity as well as region j's aggregate productivity; and the firm's ability to deploy its productivity and intangible capital abroad may be limited by FDI barriers, σ_{ij}, as in McGrattan and Waddle (2020). We assume that $\sigma_{ij} \in [0, 1]$ and that $\sigma_{ii} = 1$.

Research & development. Firms hire workers in their domestic parent corporations to produce intangible capital. We assume that labor productivity in R&D is the same as TFP in production. In other words, it takes $1/A_i$ workers in region i to produce one unit of intangible capital, i.e., the cost to produce z units of intangible capital is $W_i z/A_i$. Following McGrattan and Waddle (2020), we assume that R&D expenditures are tax-deductible.[10]

Trade and foreign direct investment. Firms can sell in the domestic market freely, but serving foreign markets is costly. There are two options for serving foreign markets: pay a fixed cost κ_i^X to export domestically produced goods; or pay a fixed cost κ_i^F to open a foreign affiliate and produce locally. Fixed costs are denominated in units of the home country's labor. Each unit of goods shipped abroad incurs an iceberg transportation cost ξ_{ij}. Firms can simultaneously export to, and produce locally for, the same foreign country; exports and locally produced products are considered distinct varieties as in Garetto et al. (2019) and McGrattan and Waddle (2020).[11] Let $J_X \subseteq I \setminus \{i\}$ denote the set of foreign regions to which a firm exports, and let $J_F \subseteq I \setminus \{i\}$ denote the set of regions in which it operates a foreign

[10] Alternatively, one could treat R&D like tangible investment, but this would require a dynamic model in which a depreciation allowance is deducted from taxes instead. We leave a dynamic treatment of profit shifting for future research.

[11] We have also studied a version of the model in which firms must choose whether to export or produce locally for each foreign market (the standard proximity-concentration trade-off), and the results are similar.

affiliate. The firm's resource constraints can then be written as follows:

$$y_{ii} = q_{ii} + \sum_{j \in J_X} \xi_{ij} q_{ij}^X \tag{35}$$

$$y_{ij} = q_{ij}, \; j \in J_F \tag{36}$$

where we distinguish exported goods, denoted as q_{ij}^X, from goods that are produced and consumed in the same location, q_{ij}.

Transfer pricing. As in section 3, foreign subsidiaries pay licensing fees to use intangible capital. As before, the licensing fee of a subsidiary in region j is given by $\vartheta_{ij} z$, where $\vartheta_{ij} \equiv \gamma p_{ij} y_{ij}/z$ is the marginal revenue product of intangible capital, and the total amount of licensing fees across the conglomerate is $\nu_i z \equiv \sum_{j \in J_F \cup \{i\}} \vartheta_{ij} z$. Note again that this includes the licensing fee for the parent corporation's use of its own intangible capital.

Profit shifting. Also as in section 3, a firm based in a high-tax region can shift its profits by transferring ownership of its intangible capital to its affiliates in lower-tax jurisdictions (provided that the firm has paid the fixed costs to establish these affiliates). Consider a firm that sells a fraction λ^{LT} of its intangible capital to the low-tax region and a fraction λ^{TH} to the tax haven. Its affiliate in the former collects licensing fees of

$$\lambda_{LT} \sum_{j \in J_F \cup \{i\} \setminus \{LT\}} \vartheta_{ij} z, \tag{37}$$

while its affiliate in the latter collects

$$\lambda_{TH} \sum_{j \in J_F \cup \{i\}} \vartheta_{ij} z. \tag{38}$$

The domestic parent collects the remaining fees:

$$(1 - \lambda_{LT} - \lambda_{TH}) \sum_{j \in J_F} \vartheta_{ij} z. \tag{39}$$

Setting up an affiliate in the tax haven requires a fixed cost κ_i^{TH}. The cost (paid in units of labor in the home country) to sell a fraction λ of intangible capital to another country is specified as $\mathcal{C}_{ij}(\lambda) = [\lambda + (1-\lambda)\log(1-\lambda)]\, \psi_{ij}$, where ψ_{ij} governs the marginal cost. The cost to sell a fraction λ_{LT} of intangible capital to the low-tax region and a fraction λ_{TH} to the tax haven is $\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH})$. Recall that in the simple theoretical model in section

3, the profit shifting technology is governed by the discount φ at which the MNE can sell its intangible capital to its affiliate in the tax haven. In our quantitative setting, the marginal cost parameter ψ_{ij} captures this discount as well as the resource cost of shifting profits. This allows for more flexibility and allows our model to generate a wider range of profit-shifting outcomes.[12,13]

4.3 The firm's problem

The firm's objective is to maximize its dividend payout. We describe the firm's problem in three steps: first, in a standard environment without transfer pricing or profit shifting; second, with transfer pricing but without profit shifting; and third, with profit shifting.

4.3.1 Free transfer scenario

Here, the firm chooses where to export (J_X); where to open a foreign affiliate (J_F); how much intangible capital to produce (z); how much labor to hire in each of its divisions (ℓ_{ij}); and how much to sell to each of its markets (q_{ij}, q_{ij}^X). We can break this problem into two stages, working backward. In the second stage, the firm maximizes each division's gross operating profits taking J_X, J_F, and z as given. The domestic parent corporation's profits are

$$\pi_i^D(a, z; J_X) = \max_{q_{ii}, \{q_{ij}^X\}_{j \in J_X}, \ell_i} \left\{ p_{ii}(q_{ii})q_{ii} + \sum_{j \in J_X} p_{ij}(q_{ij}^X)q_{ij}^X - W_i \ell_i \right\}$$
$$\text{s.t} \quad q_{ii} + \sum_{j \in J_X} \xi_{ij} q_{ij} = y_i = A_i a (N_i z)^{\gamma} \ell_i^{\phi}. \tag{40}$$

[12] ψ_{ij} enters the first-order conditions for λ and z in exactly the same way as φ, so we could not separately identify the two parameters if they were both included. For example, the solution for λ_j is now given by $\lambda_j = 1 - \exp\left(-\frac{1-\varphi}{\psi_{ij}} \frac{\tau_i - \tau_{i*}}{1-\tau_i}\right)$. We could pick any value for φ and recalibrate ψ_{ij} to match our target moments, and the equilibrium would always be identical.

[13] Strictly speaking, we assume that affiliates in the low-tax region and tax haven acquire intangible capital property rights from the parent for free (a discount of 100%, i.e., $\varphi = 0$). What matters from the MNE's perspective at the micro level is not what the affiliate pays per se, but the net effect on the MNE's global net profit, $(\tau_j - \tau_i)\varphi$, which is subsumed by the parameter ψ_{ij} in our quantitative model as described in footnote 12. Regardless, this is actually a reasonable approximation of reality. For example, according to the testimony of U.S. Senator Carl Levin, Apple's Irish subsidiary ASI "paid approximately $5 billion to Apple…[and] received profits of $74 billion," which would imply a discount of more than 93%. See Levin's full statement here: https://www.hsgac.senate.gov/imo/media/doc/OPENING%20STMT%20-%20LEVIN-Carl%20Offshore%20Profit%20Shifting%20(Apple)%205-21-13.pdf.

Foreign subsidiaries' profits are

$$\pi_{ij}^{F}(a,z) = \max_{q_{ij},\ell_j} \; p_{ij}(q_{ij})q_{ij} - W_j\ell_j, \; j \in J_F. \tag{41}$$

Note that these objects will not change when we incorporate transfer pricing and profit shifting.

In the first stage, the firm chooses J_X, J_F, and z to maximize its global net profits, taking into account the cost of producing intangible capital, as well as the fixed costs of exporting and opening foreign affiliates:

$$\begin{aligned} d_i^{FT}(a) = \max_{z,J_X,J_F} \Bigg\{ &(1-\tau_i)\Bigg[\overbrace{\pi_i^D(a,z;J_X) - W_i\Bigg(z/A_i + \sum_{j\in J_X}\kappa_{ijX} + \sum_{j\in J_F}\kappa_{ijF}\Bigg)}^{\pi_{ii}^{FT}}\Bigg] \\ &+ \sum_{j\in J_F}(1-\tau_j)\underbrace{\pi_{ij}^F(a,z)}_{\pi_{ij}^{FT}}\Bigg\}. \end{aligned} \tag{42}$$

See Appendix F.1 for more details on the solution to this problem. We use π_{ii}^{FT} and π_{ij}^{FT} to denote the firm's taxable profits in its domestic parent division and foreign subsidiaries, respectively, in this scenario.

4.3.2 Transfer pricing scenario

Here, the firm makes the same choices as in the *free transfer* scenario, but it takes into account the licensing fees that its foreign affiliates pay to the parent corporation. The first stage of the firm's problem in this scenario is

$$\begin{aligned} d_i^{TP}(a) = \max_{z,J_X,J_F} \Bigg\{ &(1-\tau_i)\Bigg[\overbrace{\pi_i^D(a,z;J_X) - W_i\Bigg(z/A_i + \sum_{j\in J_X}\kappa_{ijX} + \sum_{j\in J_F}\kappa_{ijF}\Bigg) + \sum_{j\in J_F}\vartheta_{ij}(z)z}^{\pi_{ii}^{TP}}\Bigg] \\ &+ \sum_{j\in J_F}(1-\tau_j)\Bigg[\underbrace{\pi_{ij}^F(a,z) - \vartheta_{ij}(z)z}_{\pi_{ij}^{TP}}\Bigg]\Bigg\}. \end{aligned} \tag{43}$$

We make explicit the dependence of the licensing fees on the firm's choice of intangible capital by writing $\vartheta_{ij}(z)$ as a function of z. In contrast to our simple static framework, firms in our quantitative model internalize the effects of their choices of z on transfer prices. See Appendix F.2 for more details on the solution to this version of the problem. In this scenario, π_{ii}^{TP} and π_{ij}^{TP} denote the firm's taxable profits in its domestic and foreign divisions, respectively. The difference between these objects and their counterparts in the free transfer scenario is intangible capital licensing fees, which increase taxable profits in the parent and reduce them in foreign subsidiaries. As we will see, these fees will be crucial in defining the amount of lost profits in our model with profit shifting.

4.3.3 Profit shifting scenario

Profit shifting adds an additional decision: how much intangible capital to shift to affiliates in the low-tax region and/or tax haven. This problem can be written as

$$
\begin{aligned}
d_i^{PS}(a) = \max_{z, J_X, J_F, \lambda_{TL}, \lambda_{TH}} \Bigg\{ & (1-\tau_i)\Bigg[\pi_i^D(a, z; J_X) - W_i\Big(z/A_i + \sum_{j \in J_X} \kappa_{ijX} + \sum_{j \in J_F} \kappa_{ijF}\Big) \\
& + \overbrace{\sum_{j \in J_F} (1-\lambda_{LT}-\lambda_{TH})\vartheta_{ij}(z)z}^{\text{Licensing fee receipts}} - \overbrace{(\lambda_{LT}+\lambda_{TH})\vartheta_{ii}(z)z}^{\text{Licensing fee payments}} \\
& - \overbrace{W_i \kappa_{iTH} \mathbb{1}(\lambda_{TH} > 0)}^{\text{Tax haven affiliate cost}} - \overbrace{W_i(\mathcal{C}_{i,TH}(\lambda_{TH}) + \mathcal{C}_{i,LT}(\lambda_{LT}))\nu_i(z)z}^{\text{Cost of transferring } z}\Bigg] \\
& + (1-\tau_{LT})\Bigg[\pi_{i,LT}^F(a,z) + \overbrace{\sum_{j \in J_F \cup \{i\} \setminus \{LT\}} \lambda_{LT}\vartheta_{ij}(z)z}^{\text{Licensing fee receipts}} - \underbrace{(1-\lambda_{LT})\vartheta_{iLT}(z)z}_{\text{Licensing fee payment}}\Bigg]\mathbb{1}_{\{LT \in J_F\}} \\
& + (1-\tau_{TH})\underbrace{\Bigg[\sum_{j \in J_F \cup \{i\}} \lambda_{TH}\vartheta_{ij}(z)z\Bigg]}_{\text{Licensing fee receipts}}\mathbb{1}_{\{\lambda_{TH} > 0\}} \\
& + \sum_{j \in J_F \setminus \{LT\}} (1-\tau_j)\Bigg[\pi_{ij}^F(a,z) - \underbrace{\vartheta_{ij}(z)z}_{\text{Licensing fee}}\Bigg]\Bigg\}
\end{aligned}
\tag{44}
$$

subject to $\lambda_{LT} + \lambda_{TH} \leq 1$ and $\lambda_{LT} \leq \mathbb{1}_{\{LT \in J_F\}}$. The last inequality simply says that you cannot shift profits to the low-tax region if you do not have an affiliate there. Note that firms in the low-tax region do not choose λ_{LT}, only λ_{TH}. See Appendix F.3 for more details on how to solve this problem. The first square-bracketed term represents the profits of the parent division, π_{ii}^{PS} in this scenario, the second term represents the profits of the low-tax affiliate, $\pi_{i,LT}^{PS}$, the third represents the profits of the tax-haven affiliate, $\pi_{i,TH}^{PS}$, and the fourth represents the profits of affiliates in other high-tax regions, π_{ij}^{PS}.[14]

4.4 Aggregation and accounting measures

Several national and international accounting measures are required to close the model and compare it to the data. Here, we revert to expressing firms' choices as functions of their varieties (ω) for notational brevity.

Gross domestic product. Nominal GDP is the total value of goods produced in a given region:

$$GDP_i = \sum_{j=1}^{I} \int_{\omega \in \Omega_j, i \in J_F(\omega)} p_{ji}(\omega) y_{ji}(\omega)\ d\omega. \tag{45}$$

We compute real GDP by deflating by the consumer price index P_i defined in (33).

Goods trade. Aggregate goods trade flows are given by

$$EX_i^G = \sum_{j \neq i} \int_{\Omega_i} p_{ij}^X(\omega)\left(1 + \xi_{ij}\right) q_{ij}^X(\omega)\ d\omega, \tag{46}$$

$$IM_i^G = \sum_{j \neq i} \int_{\Omega_j} p_{ji}^X(\omega)\left(1 + \xi_{ji}\right) q_{ji}^X(\omega)\ d\omega. \tag{47}$$

Services trade. As in Guvenen et al. (2022), intangible capital licensing fees enter the national accounts as exports or imports of intellectual property services. High-tax regions' services trade flows are given by

$$EX_i^S = \sum_{j \neq i} \int_{\Omega_i} \left[1 - \lambda_{LT}(\omega) - \lambda_{TH}(\omega)\right] \vartheta_{ij}(\omega) z(\omega)\ d\omega, \tag{48}$$

$$IM_i^S = \sum_{j \neq i} \int_{\Omega_i} \left[\lambda_{LT}(\omega) + \lambda_{TH}(\omega)\right] \vartheta_{ij}(\omega) z(\omega)\ d\omega + \sum_{j \neq i} \int_{\Omega_j} \vartheta_{ji}(\omega) z(\omega)\ d\omega. \tag{49}$$

[14] We abstract in our model from the Global Intangible Low Tax Income (GILTI), adopted by the U.S. government in 2017, for two reasons. First, once we take the model to the data (see next section) we treat North America as a single region. Second, according to the scarce literature on GILTI, see Clausing (2020b) and Garcia-Bernardo et al. (2022), it had limited impact on profit shifting conducted by the U.S. multinationals.

The low-tax region's services trade flows are

$$EX_{LT}^S = \sum_{j \neq i} \int_{\Omega_i} [1 - \lambda_{TH}(\omega)] \, \vartheta_{ij}(\omega) z(\omega) \; d\omega + \sum_{j \neq i} \int_{\Omega_j} \lambda_{LT} \vartheta_{ji}(\omega) z(\omega) \; d\omega, \tag{50}$$

$$IM_{LT}^S = \sum_{j \neq i} \int_{\Omega_i} \lambda_{TH}(\omega) \vartheta_{ij}(\omega) z(\omega) \; d\omega + \sum_{j \neq i} \int_{\Omega_j} [1 - \lambda_{LT}(\omega)] \vartheta_{ji}(\omega) z(\omega) \; d\omega. \tag{51}$$

Note that in the *TP* scenario, $\lambda_{LT}(\omega) = \lambda_{TH}(\omega) = 0$. We can also write the tax haven's services exports (it has no imports because foreign affiliates located there do not produce anything) as

$$EX_{TH}^S = \sum_{j=1}^{I} \int_{\Omega_j} \lambda_{TH} \vartheta_{ji}(\omega) z(\omega) \; d\omega. \tag{52}$$

Net factor receipts and payments. Net factor receipts from (payments to) foreigners are the sum total of the dividends paid by foreign subsidiaries of domestic multinationals (domestic subsidiaries of foreign multinationals):

$$NFR_i = \sum_{j \neq i} \int_{\Omega_i} (1 - \tau_j) \pi_{ij}^{PS}(\omega) \; d\omega, \tag{53}$$

$$NFP_i = \sum_{j \neq i} \int_{\Omega_j} (1 - \tau_i) \pi_{ji}^{PS}(\omega) \; d\omega. \tag{54}$$

In the *FT* and *TP* scenarios, we use π_{ij}^{FT} and π_{ij}^{TP}, respectively, to calculate these objects.

Shifted profits. We define the profits shifted out of region j by a firm ω that is based in region i by comparing the profits the firm books in j in the *PS* scenario to the profits it would book in the *TP* scenario:

$$\tilde{\pi}_{ij}(\omega) = \pi_{ij}^{TP}(\omega) - \pi_{ij}^{PS}(\omega). \tag{55}$$

When $\tilde{\pi}_{ij}(\omega) > 0$, this indicates that the firm would book more profits in region j in the absence of profit shifting, i.e., the firm has shifted profits away from region j. Aggregating shifted profits by firms at the region level yields the total profits shifted out of region j:

$$\tilde{\Pi}_j = \sum_{i=1}^{I} \int_{\Omega_i} \tilde{\pi}_{ij}(\omega) \; d\omega. \tag{56}$$

Note that $\pi_{ij}^{TP}(\omega)$ is a counterfactual object that can be computed in partial equilibrium or general equilibrium. In partial equilibrium, we calculate it while holding fixed firms' decision rules from the *PS* scenario. In general equilibrium, on the other hand, we re-solve the firm's problem for the *TP* scenario, which changes allocations at the micro level and ultimately at the macro level as well. We use the partial-equilibrium version of this measure in our calibration procedure, but we use the general-equilibrium version when analyzing the implications of the two pillars of the OECD proposal.

4.5 Market clearing and equilibrium

In a general equilibrium of our model, the labor market must clear, the government's budget constraint must be satisfied, and the balance of payments must hold in each productive region.

Labor market. Labor demand comes from four sources: production of intermediate goods; production of intangible capital; fixed costs of exporting and setting up foreign affiliates; and the costs of transferring intangible capital. The labor market clearing condition can be written as

$$L_i = \overbrace{\sum_{j=1}^{I} \int_{\Omega_j} \ell_{ji}(\omega)\ d\omega}^{\text{goods production}} + \overbrace{\int_{\Omega_i} z(\omega)/A_i\ d\omega}^{z\text{ production}} + \overbrace{\int_{\Omega_i} \left(\sum_{j\in J_X(\omega)} \kappa_i^X + \sum_{j\in J_F(\omega)} \kappa_i^F + \mathbb{1}_{\{\lambda_{TH}(\omega)>0\}} \kappa_i^{TH} \right) d\omega}^{\text{fixed costs}}$$
$$+ \underbrace{\int_{\Omega_i} \left(\mathcal{C}_{i,TH}(\lambda_{TH}) + \mathcal{C}_{i,LT}(\lambda_{LT})\right) \nu(\omega) z(\omega)\ d\omega}_{\text{costs of shifting } z}. \tag{57}$$

Note that at the macro level, profit shifting diverts labor from goods production and R&D to wasteful administrative costs, potentially offsetting the positive macroeconomic effects of increased R&D at the micro level.

Government budget constraint. We assume that revenue from corporate income taxation is rebated lump-sum to households.[15] In the benchmark *PS* model, lump-sum transfers are given by

$$T_i = \tau_i \sum_{j=1}^{I} \int_{\Omega_j} \pi_{ji}^{PS}(\omega)\ d\omega. \tag{58}$$

In the *FT* and *TP* scenarios, π_{ji}^{FT} and π_{ji}^{TP} are used instead.

[15] We have also analyzed a version of the model in which labor income taxes adjust to clear the government's budget constraint, and the results are similar.

Balance of payments. The balance of payments requires that each region's current account must be zero:

$$EX_i^G + EX_i^S - IM_i^G - IM_i^S + NFR_i - NFP_i = 0. \tag{59}$$

Note that several things happen to the balance of payments when a firm shifts profits away from its home region. First, that region's services trade balance worsens: the firm receives fewer licensing fees from its foreign subsidiaries and makes more licensing payments. Second, net factor receipts rise: the firm's profits in the tax haven (or low-tax region) rise, and these increased profits are ultimately rebated back to the home country. These two effects offset one another, but not completely: some of the shifted profits are taxed and therefore remain in the tax haven and/or low-tax region. Thus, the net effect is that the current account worsens.[16] To regain equilibrium, that trade balance and/or net factor income balance must improve, which shows up in our model as a real exchange rate depreciation.

Competitive equilibrium. Given a set of parameters and a scenario (FT, TP, or PS), an equilibrium in our model is a set of aggregate prices and quantities $\{W_i, P_i, C_i, L_i\}$ and a set of firm decision rules $\{J_X(\omega), J_F(\omega), z(\omega), \boldsymbol{\ell}(\omega), \boldsymbol{q}(\omega), \lambda_{LT}(\omega), \lambda_{TH}(\omega)\}$ for each productive region $i \in J$ that satisfy

1. the representative household's utility maximization problem described by (29)–(33);
2. the firm's profit-maximization problem described by (40), (41), and either (42), (43), or (44);
3. the labor-market clearing condition (57);
4. the government's budget constraint (58); and
5. the balance of payments (59).

4.6 Calibration

We calibrate our model's parameters so that its equilibrium, given the current international tax regime, reproduces salient facts about production, international trade, foreign direct investment, and, most importantly, profit shifting. Some of the parameters, like elasticities of substitution, are assigned externally to standard values, while others, like population, can be set directly to exact data analogues. The remaining parameters are jointly calibrated by

[16] The reduction in the services trade balance and the increase in net factor income are consistent with the accounting of Guvenen et al. (2022). The net negative effect on the balance of payments is consistent with the findings of Hebous et al. (2021).

matching a set of target moments. These parameters influence all of the target moments to some degree, but there is one target that provides most of the identification for each parameter. Thus, in what follows, we describe each calibrated parameter alongside its main target. Table 1 lists each parameter in our model alongside its source or target moment. Table 2 provides more detailed information about region-specific target moments and parameter values. Appendix B provides details on the data sources we use to discipline the model.

Regions. We partition the world into five regions. The countries identified as tax havens by Tørsløv et al. (2022) are split into two regions: a low-tax productive region, LT, including Belgium, Ireland, Hong Kong, the Netherlands, Singapore, and Switzerland; and an unproductive tax-haven region, TH, including Luxembourg, small European countries and territories like Cyprus, Malta, and the Isle of Man, and a number of Carribean countries.[17] The other three regions are North America, Europe (except for the countries in the low-tax and tax-haven regions), and the rest of the world. Data for each region are obtained by aggregating or averaging country-level data.

Assigned parameters. The elasticity of substitution between varieties, ρ, is set to the standard value of 5. Each region's population, N_i, is set by aggregating country-level data from the World Bank's World Development Indicators database. Corporate income tax rates, τ_i, are set by averaging country-level estimates of effective corporate income tax rates from Tørsløv et al. (2022).

Technology capital share (ϕ). We set the technology capital share in the production function (34) to match the share of foreign-owned firms' income that accrues to intangible capital, which is estimated by Cadestin et al. (2021) to be 28%. Note that domestic-owned firms have lower intangible income shares, at around 22%. Although we do not target this moment in our calibration, our model is consistent with this fact. This is because technology capital is nonrival, which means that multinational firms have a greater incentive to invest in it than non-MNEs.

Total factor productivity (A_i). Each region's TFP is set to match its aggregated real GDP based on PPP-adjusted data from the World Development Indicators database.

[17] The complete list of countries in the tax-haven region is: Andorra, Anguilla, Antigua, Aruba, the Bahamas, Bahrain, Barbados, Belize, Bermuda, British Virgin Islands, Cayman Islands, Curacao, Cyprus, Gibraltar, Grenada, Guernsey, the Isle of Man, Jersey, Lebanon, Liechtenstein, Luxembourg, Malta, Marshall Islands, Mauritius, Monaco, the Netherlands Antilles, Panama, Puerto Rico, Samoa, Seychelles, Sint Maartin, St. Kitts & Nevis, St. Vincent & the Grenadines, St. Lucia, the Turks & Caicos, and Vanuatu.

Productivity distribution ($F_i(a)$). We assume that firms' productivities are drawn from Pareto distributions with region-specific tail parameters η_i. We calibrate these tail parameters to match the share of aggregate employment that is accounted for by firms with fewer than 100 times the average number of employees, which is equal to 58.9% in data published by the U.S. Census Bureau. Although this is the only moment of the firm-size distribution that we target, our model's Lorenz curve is very close to its empirical counterpart.

Utility weight on leisure (ψ_i). We choose the weight on leisure in the utility function (29) so that the representative household in each country works for one-third of its time endowment, i.e., $L_i = N_i/3$.

Variable trade cost (ξ_{ij}). We set the iceberg trade barriers to match aggregate bilateral imports of goods (agriculture, resource extraction, and manufacturing) relative to nominal GDP. Import data are from the World Input Output Database. Nominal GDP data are from the World Development Indicators. For both, we sum across the countries within each region.

Fixed export cost (κ_i^X). Each region's fixed cost of exporting is chosen so that 22.7% of firms export, as reported by Alessandria et al. (2021).

Variable FDI cost (σ_{ij}). We calibrate the parameters that govern the efficiency with which technology capital can be deployed abroad to match the share of each region's gross value added that is accounted for by foreign multinationals. These data come from the OECD AMNE database. This share is equal to 11.12% in North America, 19.82% in Europe, 28.74% in the low-tax region, and 9.55% in the rest of the world.

Fixed FDI cost to productive regions (κ_i^F). The fixed costs of establishing foreign affiliates in other productive regions are set to match the average employment of multinational firms (i.e., firms with foreign affiliates) relative to the overall average employment of all firms. This ratio is equal to 444. The former is calculated using Compustat, while the latter is calculated using data from the U.S. Census.[18]

Variable profit-shifting costs (ψ_{iLT}, ψ_{iTH}). The parameters that govern the cost of transferring technology capital are calibrated by matching Tørsløv et al. (2022)'s estimates of (i) total lost profits, and (ii) the share of lost profits that are shifted to countries in our tax-haven region. As with production and trade data, we obtain region-level measures by summing the

[18]Compustat contains data on public firms only. We do not have information on employment of private multinational firms. Our approach assumes that private multinationals are similar in size to public multinationals.

country-level estimates reported in this paper. Total lost profits are \$143bn for North America, \$216bn for Europe, and \$257bn for the rest of the world. The shares of these totals that are shifted to the tax-haven region are 66.39%, 44.50%, and 71.69%, respectively.

Fixed FDI cost to tax haven (κ_i^{TH}). The fixed costs of establishing affiliates in the tax haven region are set to match the average employment of firms that have affiliates in at least one country in our tax haven region. This ratio is equal to 981. It is also calculated using Compustat.

4.7 External validation

We have calibrated the key parameters of our model—the profit-shifting costs, ψ_{ij}—to match macroeconomic estimates of aggregate lost profits. However, our calibrated model also matches microeconomic estimates of firm-level profit shifting very closely. We discuss the empirical literature on this topic in detail in Appendix C. The key object of interest in this literature is the semi-elasticity of reported pre-tax profits in an MNE's domestic parent division with respect to the tax differential between the home country and a foreign tax haven. In Table 3 we report three semi-elasticities estimated by key studies in the empirical literature. They range from 0.8 to 1.1, which means that a one-percentage-point decrease in the tax rate differential—for example, as a consequence of an increase in tax-haven's tax rate—is associated with a 0.8% to 1.1% increase in pre-tax profits reported at home.

To obtain a model counterpart of these elasticities, we estimate the following specification on simulated data generated from counterfactual experiments in which we perturb the different regions' tax rates:

$$\log \pi_i^{k,PS}(\omega) = \beta_0 + \beta_\ell \log \ell_i^k(\omega) + \beta_z \log z^k(\omega) - \beta_\tau \hat{\tau}_i^k + \epsilon_i^k(\omega), \tag{60}$$

where k denotes the index of the counterfactual economy and $\hat{\tau}_i^k$ denotes the tax differential between an MNE's home region and the profit-shifting destination region (either the low-tax region or the tax haven). The parameter of interest is β_τ, which is the relevant semi-elasticity in our model. Appendix C.3 contains more details on how we produce the model-generated data and specify the empirical regression. As Table 3 shows, we obtain an estimate of $\beta_\tau = 0.87$, which lies comfortably within the narrow bounds of the estimates in the empirical literature.

The fact that our calibrated quantitative model is consistent with the microeconomic evidence on profit shifting as well the macroeconomic evidence indicates that it is well suited to measuring the macroeconomic effects of profit shifting and two-pillar OECD/G20 reform.

5 Quantitative Results

Having described the model and its calibration, we turn now to the results of our quantitative analysis. First, we illustrate the effects of transfer pricing and profit shifting by comparing our baseline model to counterfactuals without these ingredients. Second, we analyze the effects of the two pillars of the OECD BEPS project.

5.1 Inspecting the mechanism

Before using the calibrated model to analyze the consequences of changing the global corporate income tax landscape, it is helpful to illustrate the effects of our model's novel ingredients—transfer pricing and profit shifting—under the current tax system. We do this by comparing our baseline model, in which MNEs license technology capital to foreign affiliates according to transfer pricing rules and shift profits by selling technology capital to their affiliates in the tax haven, to two counterfactual models. In the first, the domestic parent corporation retains ownership of technology capital but still licenses this capital to foreign affiliates according to the arm's length principle. We refer to this version as the *no-shifting* counterfactual. In the second, the cost of foreign affiliates' usage of technology capital is not accounted for at all (licensing fees are set to zero). We refer to this version as the *no-transfer-pricing* counterfactual.

Effects of transfer pricing. To illustrate the effects of transfer pricing, panel (a) of Table 4 shows how the no-shifting counterfactual compares to the no-transfer-pricing counterfactual. In the highest-tax region in our model, North America, MNEs reduce R&D and produce less output, consistent with part 1 of Proposition 1. In other regions, however, MNEs' R&D actually increases. North America, as a large, high-productivity region, is an important FDI destination for these other regions' MNEs. Transfer pricing allows these MNEs to book the returns to intangible capital in their North American subsidaries, which face the highest tax rates, as profits in their domestic parent divisions, which face lower tax rates. This effect is most pronounced in the low-tax region; this is effectively the reverse of part 1 of Proposition 1. In this case, there is also a notable general equilibrium effect for non-MNEs that operates in the opposite direction: greater labor demand by MNEs increases prices, crowding out non-MNEs.

Although the effects of transfer pricing on R&D differ across regions, output falls in equilibrium everywhere, albeit for different reasons. In North America, the decline in output is driven by the response of domestic MNEs. Note that output of foreign MNEs' North

American subsidiaries actually rises, but because foreign MNEs account for a relatively small share of overall North American output, this increase is not enough to offset the decline in domestic firms' output. In other regions, the output decline is driven primarily by foreign MNEs, specifically those from North America whose R&D falls.

The effects on corporate tax revenues are heterogeneous across regions. Revenues rise in high-tax North America because licensing fees reallocate income from domestic MNEs' foreign subsidiaries to their parent divisions. In Europe and the rest of the world, revenues fall for the opposite reason: profits of foreign MNEs' subsidiaries in these regions fall when they must pay to use intangible capital. In the low-tax region, revenues rise because domestic MNEs do more R&D and earn more profits globally, which return home in the form of licensing fees.

Effects of profit shifting. Panel (b) of Table 4 demonstrates the effects of profit shifting by comparing the baseline model to the no-shifting counterfactual. These effects are easier to explain, as they are the same in the three high-tax regions, North America, Europe, and the rest of the world. In these regions, MNEs increase R&D and produce more output, consistent with part 2 of Proposition 1, and this ultimately leads to higher aggregate output. At the same time, profit shifting reduces corporate tax revenues, with the largest effect in Europe.

In the low-tax region, profits shifted in from the high-tax regions amount to almost 4 percent of GDP and tax revenues rise by a full 23.5 percent. In equilibrium, this increase in income raises prices, reducing R&D among both MNEs and non-MNEs. However, the effect of this reduction on aggregate output is offset to a large degree by higher production by foreign MNEs' subsidiaries in this region.

5.2 Policy experiments

We use our calibrated model to conduct four experiments to analyze the macroeconomic consequences of the policies proposed in the OECD/G20 Inclusive Framework on BEPS described in detail in Appendix A.2. In the first experiment we focus on the first pillar of this framework, which allocates a portion of an MNE's overall global profit to its subsidiaries based on these subsidiaries' revenues. The second experiment focuses on the second pillar, which imposes a global minimum corporate income tax rate of 15 percent. In the third experiment, we analyze the combined effects of these two pillars together. In the fourth experiment (which is really a set of sub-experiments) we study the combined effects of both pillars under different values for the profit reallocation share and global minimum tax rate. In all four experiments, we restrict attention to long-run analysis, comparing the steady state under the current regime to the steady state after the policy is implemented. Table 5 and

Figure 1 show the results of these experiments. Appendix D shows that our main results are robust to to a wide range of alternative setups and calibrations, such as different intangible capital shares and profit-shifting costs.

OECD Pillar 1: revenue-based profit allocation. As described in Appendix A.2, the first pillar of the OECD BEPS project allocates, for the purposes of taxation, a fraction of a firm's global profits to the countries in which the firm sells its products. Following the OECD proposal, this allocation is based on these countries' shares of the firm's overall global sales. Importantly, it is independent of whether the firm has a physical presence in these countries, which implies that non-MNE exporters are also subject to this rule. The firm's problem under this rule can be written as

$$d_i^{TP}(a) = \max_{z, J_X, J_F} \Bigg\{ \pi_i^D(a, z; J_X) - W_i\Bigg(z/A_i + \sum_{J \in J_X} \kappa_{ijX} + \sum_{j \in J_F} \kappa_{ijF} \Bigg) + \sum_{j \in J_F} \vartheta_{ij}(z) z \\ + \sum_{j \in J_F} \Big[\pi_{ij}^F(a, z) - \vartheta_{ij}(z) z \Big] - \sum_{j \in J_F \cup J_X \cup \{i\}} \tau_j T_{ij}(a, z) \Bigg\}, \tag{61}$$

where $T_{ij}(a, z)$ represents the tax base for region j under the profit allocation rule. In Appendix F.4, we show that $T_{ij}(a, z)$ is given by

$$T_{ii}(a, z) = (1 - \theta) \cdot \pi_i^D(a, z; J_X) + \theta \cdot \hat{S}_{ii}(a, z) \cdot \Bigg[\pi_i^D(a, z; J_X) + \sum_{j \in J_F} \pi_j^F(a, z) \Bigg], \tag{62}$$

$$T_{ij}(a, z) = (1 - \theta) \cdot \pi_{ij}^F(a, z) + \theta \cdot \hat{S}_{ij}(a, z) \cdot \Bigg[\pi_i^D(a, z; J_X) + \sum_{j \in J_F} \pi_j^F(a, z) \Bigg], \ j \in J_F, \tag{63}$$

$$T_{ij}(a, z) = \theta \cdot \hat{S}_{ij}(a, z) \cdot \Bigg[\pi_i^D(a, z; J_X) + \sum_{j \in J_F} \pi_j^F(a, z) \Bigg], \ j \in J_X \setminus \{J_F\} \tag{64}$$

where θ is the fraction of residual profits that are reallocated and

$$\hat{S}_{ij}(a, z) = \frac{p_{ij}(a, z) q_{ij}(a, z) + p_{ij}^X(a, z) q_{ij}^X(a, z)}{\sum_{k \in \{i\} \cup J_X \cup J_F} \left[p_{ik}(a, z) q_{ik}(a, z) + p_{ik}^X(a, z) q_{ik}^X(a, z) \right]} \tag{65}$$

is region j's share of the firm's total global sales. The OECD's proposal sets θ to 25%.

Panel (a) of Table 5 shows the effects of this pillar. It would indeed make a large dent in international profit shifting and materially raise high-tax countries' corporate income tax revenues. Lost profits would fall by 34–40% in North America, Europe, and the rest of the world, and tax revenues would increase by 1.6–2.6%. In the low-tax region, profits shifted inward would fall by 31% and tax revenues would fall by 11.4%. At the same time, however, this pillar would decrease output globally. MNEs based in all three high-tax regions would reduce R&D and produce less output, and although non-MNEs would expand slightly in equilibrium, overall output in these regions would decline. The effects would be largest in North America, where MNEs' R&D would fall by 0.8% and aggregate output would fall by 0.13%. In the low-tax region, domestic MNEs would increase R&D, but the decline in foreign MNEs' output in this region would ultimately drag overall output downward as well.

OECD Pillar 2: Global minimum corporate income tax. The second pillar is a global minimum corporate income tax. Following the OECD guidance, we implement this policy through top-up taxes levied by the governments of MNEs' home countries. Specifically, if a firm based in jurisdiction i reports profits in a jurisdiction j where the tax rate is below the global minimum tax rate $\underline{\tau}$, such profits are taxed in jurisdiction i at a rate equal to the tax differential, $\underline{\tau} - \tau_j$. Thus, the additional tax revenue for jurisdiction i is then

$$\tilde{T}_i = \sum_{j=1}^{I} \int_{\Omega_i} \max\left(\left(\underline{\tau} - \tau_j\right), 0\right) \pi_{ij}^{PS}(\omega)\ d\omega. \tag{66}$$

and then the adjusted budget constraint of the government becomes

$$T_i = \tau_i \sum_{j=1}^{I} \int_{\Omega_j} \pi_{ji}^{PS}(\omega)\ d\omega + \tilde{T}_i. \tag{67}$$

The rest of the equilibrium conditions stay unchanged. Panel (b) of Table 5 shows the effects of the second pillar. This policy has even larger effects on high-tax countries' lost profits and tax revenues than the first pillar. Lost profits in North America, Europe, and the rest of the world would fall by 63–85% and tax revenues would rise by 2.6–4.9%. On the other hand, the macroeconomic effects would be smaller. Although European MNEs and MNEs from the rest of the world would reduce R&D by more, North American MNEs' R&D would fall less, and low-tax MNEs' R&D would rise more. The net effect would be negligible effects on GDP in all four regions. Finally, note that in the low-tax region, profits shifted inward from the high-tax regions would fall by 51% and corporate income tax revenues would fall by 9.7%, but there would be little effect on aggregate output. This is due to the fact that while domestic firms would actually increase R&D slightly, output produced by foreign MNEs in this region would fall.

Both pillars combined. Panel (c) of Table 5 shows the effects of implementing both pillars simultaneously. Consistent with Proposition 2, the effects are larger than in either of the first two experiments, but not much so. Profit shifting would be mostly eliminated: lost profits would fall by 77% in North America, 82% in Europe, and 90% in the rest of the world. Corporate income tax revenues would rise more than under either pillar alone, especially in North America. In the low-tax region, profits shifted inward would fall by 67% and tax revenues by 16.5%. The macroeconomic effects would be slightly larger than under Pillar 1 in all regions.

Varying the reallocation share and minimum tax rate. Figure 1 shows how the effects of the two pillars change when their parameters are varied. The x-axis in each plot is Pillar 1's profit reallocation share and the y-axis is Pillar 2's global minimum tax rate. The first column of plots in the figure shows how the effects on lost profits change and the second column shows how the effects on output change. In both columns, darker shades of red indicate "worse" outcomes: smaller reductions in lost profits in the first column and larger output losses in the second column. The results of this analysis clearly show that a global minimum tax rate is better policy than profit reallocation. Both pillars are effective at reducing profit shifting, but profit reallocation causes much larger output losses. A 17 percent minimum tax rate would essentially eliminate profit shifting entirely but would not reduce output much more than the benchmark 15 percent rate. It would take a profit reallocation share of 90 percent or greater to achieve the same reduction in lost profits, but the output losses from this policy would be an order of magnitude greater.

Why does Pillar 1 have larger macroeconomic consequences? Our results show that either pillar of the OECD/G20 Framework would reduce profit shifting substantially, but Pillar 1 would have larger effects on output. The explanation for this is that Pillar 1 affects firms that do not shift profits—and even some firms that do not engage in multinational production at all—while Pillar 2 only affects MNEs that actually shift profits. Specifically, Pillar 1 allocates taxation rights based on where firms make their sales, including export markets. This aspect of the rule increases effective tax rates for firms in Europe, the low-tax region, and the rest of the world because North America, the largest, richest export market, has the highest tax rate. In Appendix D, we show that a version of Pillar 1 that allocates taxation rights based on production instead of sales would achieve the same reduction in profit shifting at a lower macroeconomic cost in these regions. In the case of North America, Pillar 1 has larger macroeconomic effects because the home share of revenue is larger than the home share of profits for firms based in this region, as intangible investment expenses (as well as the fixed costs of selling abroad) are incurred at home. Here, the alternative production-based version of Pillar 1 analyzed in the appendix has similar effects as the sales-based version.

6 Conclusion

We have developed a theory of international profit shifting by multinational enterprises (MNEs) to study the macroeconomic implications of this phenomenon. In our model, MNEs invest in nonrival intangible capital which they can use simultaneously in all of their divisions around the world. MNEs charge their foreign affiliates licensing fees to use intangible capital according to transfer pricing rules, and they can shift profits by transferring the rights to this capital to affiliates in low-tax jurisdictions.

In addition to the methodological contribution that our theory represents, we make two substantive contributions. First, we prove that profit shifting presents a trade-off between economic performance and tax revenues. On the one hand, profit shifting erodes the corporate income tax base in the jurisdiction in which an MNE is based. On the other hand, it incentivizes MNEs to invest in more intangible capital, which boosts output at home as well as abroad. Second, we calibrate our model to match empirical facts about profit shifting under the current international tax regime and use it to quantify the impact of the OECD's plan to eliminate profit shifting. This plan features two pillars: taxing MNEs in the countries in which they sell their products rather than the countries in which they book their profits; and a global minimum corporate income tax rate. We find that this reform would indeed largely eliminate profit shifting and boost tax revenues in high-tax jurisdictions. However, it would also materially reduce intangible capital investment and overall macroeconomic performance.

To put our quantitative results in context, it is helpful to compare them to the effects of other major international policy changes that have been analyzed elsewhere in the literature. Caliendo and Parro (2014) estimate that the North American Free Trade Agreement increased welfare by 0.08% in the United States and reduced it by 0.06% in Canada, while di Giovanni et al. (2014) find that the average country gained 0.13% from liberalizing trade with China. Caliendo et al. (2021) find that the 2004 EU enlargement, which liberalized international labor markets as well as trade, increased welfare in the original EU member states by 0.04%. Despite the small number of firms involved in profit-shifting—far fewer firms engage in multinational production than trade, and only a small fraction of the former shift profits—we find that the macroeconomic effects of the OECD/G20 BEPS framework would be even larger than these examples.

We have purposefully left out several aspects of profit shifting and numerous details of the proposed OECD/G20 reform in order to focus on the economic mechanisms at the core of the issue. For example, we have abstracted from manipulation of MNEs' external and internal debt (and the associated interest payments); from optimization of transfer prices; and from the proposed tax rules governing intra-company tax-deducted payments. Also, we have deliberately studied a static economy in which the international tax system gives rise only to intratemporal distortions. Thus, we view our results as a lower bound; in a dynamic model, corporate taxes also distort the intertemporal margin. We leave all these important considerations for future research.

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Table 1: Calibration overview

Parameter	Description	Value(s)	Target/source
(a) Assigned parameters			
ϱ	EoS between products	5	Standard
N_j	Population	Varies	World Development Indicators
τ_j	Corporate income tax rate	Varies	Tørsløv, Wier, and Zucman (2022)
(b) Calibrated parameters			
ϕ	Technology capital share	0.11	MNEs' intangible income share
A_i	Total factor productivity	Varies	Real GDP
η_i	Productivity dispersion	Varies	Large firms' employment share
ψ_i	Utility weight on leisure	Varies	$L_i = N_i/3$
ξ_{ij}	Variable export cost	Varies	Bilateral imports/GDP
κ_i^X	Fixed export cost	Varies	Pct. of firms that export
σ_i	Variable FDI cost	Varies	Foreign MNEs' share of value added
κ_i^F	Fixed FDI cost	Varies	Avg. emp. of firms w/ foreign affiliates
ψ_{iLT}	Cost of shifting profits to LT	Varies	Total lost profits
ψ_{iTH}	Cost of shifting profits to TH	Varies	Share of profits shifted to TH
κ_i^{TH}	Fixed cost of TH affiliate	Varies	Avg. emp. of firms w/ TH affiliates

Table 2: Calibration details

Region	North America	Europe	Low-tax	RoW	Tax haven
(a) Region-specific target moments					
Population (NA = 100)	100	92	11	1,323	–
Real GDP (NA = 100)	100	80.78	14.57	297.10	–
Corporate tax rate (%)	22.5	17.3	11.4	17.4	3.3
Foreign MNEs' VA share (%)	11.12	19.82	28.73	9.55	–
Total lost profits ($B)	143	216	–	257	–
Lost profits to TH (%)	66.4	44.5	–	71.1	–
Imports from... (% GDP)					
North America	–	1.28	1.77	1.74	–
Europe	1.70	–	12.39	3.78	–
Low tax	0.35	2.98	–	0.59	–
Row	6.15	7.96	6.78	–	–
(b) Internally calibrated parameter values					
TFP (A_i)	1.00	0.89	1.58	0.20	–
Prod. dispersion (η_i)	4.28	4.31	4.83	4.12	–
Utility weight on leisure (ψ_i)	1.06	1.08	1.09	1.06	–
Fixed export cost (κ_i^X)	1.7e-3	3.5e-3	1.0e-3	1.4e-2	–
Variable FDI cost (σ_i)	0.47	0.56	0.52	0.53	–
Fixed FDI cost (κ_i^F)	1.80	1.59	0.46	8.75	–
Cost of shifting profits to LT (ψ_{iLT})	3.40	0.38	–	2.35	–
Cost of shifting profits to TH (ψ_{iTH})	2.25	1.25	–	1.76	–
Fixed FDI cost to TH (κ_i^{TH})	0.09	0.06	–	0.59	–
Variable trade cost from...					
North America	–	3.21	3.41	2.07	–
Europe	1.89	–	1.69	1.33	–
Low tax	2.04	1.59	–	1.56	–
RoW	2.26	2.59	3.01	–	–

Notes: Population and real GDP from World Bank WDI. Corporate tax rate from Tørsløv et al. (2022). Foreign MNEs' VA share from OECD AMNE database. Fractions of firms with foreign affiliates from Compustat. Lost profits from Tørsløv et al. (2022). Imports/GDP from WIOD. Dashes (–) represent "not applicable."

Table 3: Firm-level semi-elasticity of the profit-shifting: model vs. data

Study	Data source	Headline pointstimate
Johansson et al. (2017)	ORBIS, 2000–2010	1.11
Heckemeyer and Overesch (2017)	Meta data: 27 studies, 203 estimates	0.79
Beer et al. (2020)	Meta data: 38 studies, 402 estimates	0.98
This paper	Simulated model data	0.87

Notes: The semi-elasticity of profit shifting represents the effect of a one-percentage-point decrease in the tax rate differential—for example, as a consequence of an increase in the tax haven's tax rate—on the log of pre-tax profits. For Johansson et al. (2017), we report the estimate based on their Table 1. A one-percentage-point tax difference is associated with a 0.069-percentage-point reduction in the profit-to-assets ratio (Table 1, column 1). The average MNE in the sample has a profit-to-assets ratio of 6.2%. Thus, the effect corresponds to a reduction in profits of 0.069/6.2% ≈ 1.11 (see their footnote 31). For Heckemeyer and Overesch (2017), we report the consensus estimate provided in their Table 3. For Beer et al. (2020), we report the preferred estimate provided in column 4 of their Table 2. Refer to Appendix C.3 for details of our implementation of the model estimate.

Table 4: Inspecting the mechanism

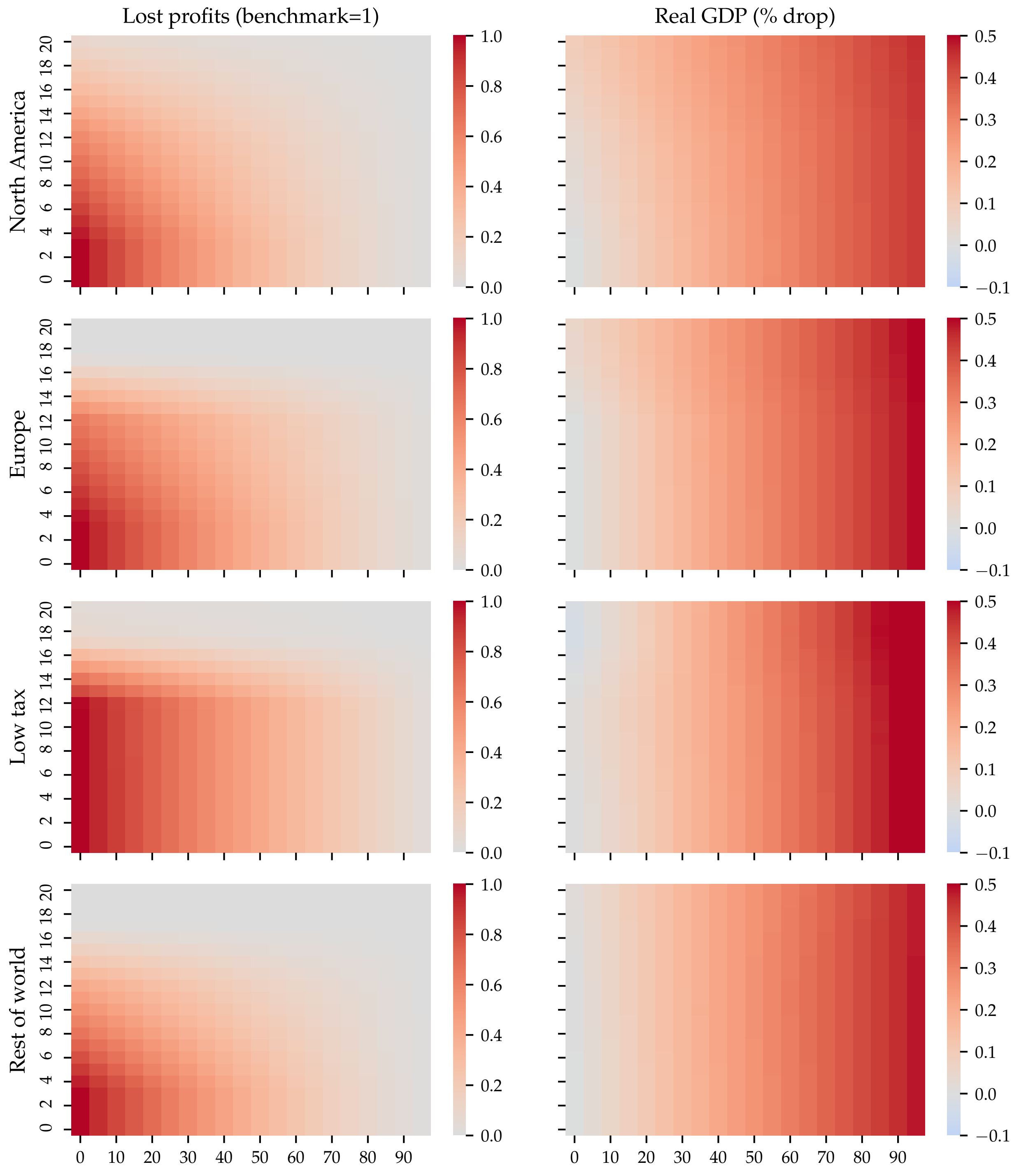
Region	Lost profits (% GDP)	Corp. tax rev. (% chg.)	Value added (% chg.)				Tech. capital (% chg.)		
			Total	Non MNEs	Domestic MNEs	Foreign MNEs	Total	Non MNEs	Domestic MNEs
(a) Effects of transfer pricing (no transfer pricing vs. no shifting)									
North America	0.00	4.32	-0.16	0.36	-0.85	0.35	-0.54	0.58	-1.34
Europe	0.00	-2.34	-0.17	-0.15	-0.11	-0.31	0.12	0.06	0.17
Low tax	0.00	-2.17	-0.25	-0.72	1.10	-0.56	0.74	-0.75	2.28
Rest of world	0.00	-0.41	-0.18	-0.18	-0.15	-0.31	0.05	0.00	0.08
(b) Effects of profit shifting (no shifting vs. baseline)									
North America	0.68	-3.82	0.08	-0.00	0.15	0.15	0.21	-0.11	0.45
Europe	1.05	-5.43	0.06	-0.02	0.17	0.04	0.26	-0.07	0.55
Low tax	-4.37	23.52	-0.04	-0.33	-0.29	0.64	-0.55	-0.60	-0.49
Rest of world	0.50	-2.59	0.04	-0.01	0.08	0.08	0.12	-0.06	0.27

Table 5: Effects of OECD BEPS pillars

Region	Lost profits (benchmark = 1)	Corp. tax rev. (% chg.)	Value added (% chg.)				Tech. capital (% chg.)		
			Total	Non MNEs	Domestic MNEs	Foreign MNEs	Total	Non MNEs	Domestic MNEs
(a) Pillar 1: Profit reallocation									
North America	0.60	2.54	-0.13	-0.01	-0.30	-0.05	-0.40	0.15	-0.80
Europe	0.66	2.61	-0.14	-0.10	-0.18	-0.17	-0.10	0.04	-0.21
Low tax	0.69	-11.40	-0.13	-0.10	0.36	-0.56	0.79	0.23	1.35
Rest of world	0.63	1.63	-0.13	-0.11	-0.15	-0.19	-0.05	0.02	-0.10
(b) Pillar 2: Global minimum tax rate									
North America	0.37	3.24	-0.06	0.01	-0.10	-0.13	-0.15	0.08	-0.31
Europe	0.26	4.89	-0.02	0.04	-0.11	-0.01	-0.22	0.06	-0.45
Low tax	0.49	-9.70	0.02	0.23	0.19	-0.46	0.32	0.36	0.28
Rest of world	0.15	2.64	-0.01	0.04	-0.05	-0.04	-0.11	0.06	-0.24
(c) Pillars 1 & 2 together									
North America	0.23	4.36	-0.17	-0.02	-0.36	-0.11	-0.48	0.17	-0.94
Europe	0.18	5.43	-0.16	-0.09	-0.24	-0.18	-0.21	0.06	-0.43
Low tax	0.33	-16.46	-0.13	0.07	0.50	-0.98	1.00	0.48	1.51
Rest of world	0.10	3.20	-0.14	-0.09	-0.17	-0.21	-0.10	0.05	-0.22

Notes: Lost profits are measured relative to the benchmark. Note that for the low-tax region, lost profits are negative in both the benchmark equilibrium and in the policy counterfactuals, i.e., profits are shifted inward to the low-tax region. However, the magnitude of these lost profits are smaller in the counterfactuals. For example, in panel (b), the amount of profits shifted into the low-tax region under pillar 2 is about half of the amount in the benchmark.

Figure 1: Varying the sizes of the pillars



Notes: Each column reports effects on one variable for each region. First column: Lost profits (reported relative to the benchmark equilibrium). Second column: real GDP (reported as a percent change from the benchmark equilibrium). X-axis in each plot represents the reallocation share for Pillar 1. Y-axis in each plot represents the global minimum corporate income tax rate for Pillar 2.

Appendix

A Institutional Background

In this section we provide a brief overview of the current international tax regime and describe the main features of the two-pillar reform proposed by the OECD. We aim here to deliver an executive summary, rather than an exhaustive discussion, of these immensely complex issues.[19] Understanding the main components of the international tax architecture is crucial since they largely dictate the setup of our theory and impose restrictions on what any reform proposal can achieve.

A.1 The Current International Tax Regime

Existing international law entitles a country to tax persons, either natural or legal, with which it has sufficient ties. In practice, taxing rights are a product of multiple national laws and international treaties that often contradict one another. The following are the most important characteristics of the current regime.

Legal separation of entities. The current regime treats subsidiaries within one MNE as separate legal entities. Thus, any transaction between parts of an MNE in different tax jurisdictions, such as for example an asset purchase, has real tax consequences. This characteristic coupled with heterogeneity of the tax systems across jurisdictions and manipulation of transfer prices gives rise to profit-shifting opportunities.

Allocation of taxing rights. There are at least four possible locations where a multinational company might in principle be taxed: the location of its shareholders, parent companies, affiliates, or customers. According to the current regime MNEs are taxed primarily in the third location (affiliates' location), but sometimes also in the second. This is achieved by a combination of legal rules allowing the countries concerned to tax on to a source or residence basis.[20]

Transfer prices. Within-MNE transactions occur at transfer prices, which are disciplined by the so-called *arm's length principle (ALP)*. The basic idea behind the ALP is that within-MNE prices should reflect the market prices that would have been charged by two independent parties of transactions. There are five core methods to achieve the ALP standard: the comparable uncontrolled price (CUP), resale price minus, cost plus, profit split, and transactional net margin methods. The practical implementation of this principle is challenging and requires complex guidelines published regularly by the OECD which member countries should obey—see OECD (2022) for the latest guide.

Treatment of intangibles. The method preferred by the OECD to implement ALP is CUP, which simply employs the price charged on comparable transactions between independent parties. CUP however is hard to implement in case of trading intangibles, since most of the time a comparable transaction is non-existent. In such cases the preferred method is the profit split method, which essentially inspects the relative financial or

[19] Our summary is largely based on Devereux et al. (2021), OECD (2015), OECD (2017), and OECD (2022).

[20] From a legal perspective, a country taxes on a residence basis when it taxes companies that are resident in that country for tax purposes on income arising in that or in another country. A country taxes on a source basis when it taxes companies that are not resident in that country for tax purposes on income deemed to arise in that country. For a thorough discussion of these concepts see Devereux et al. (2021).

other contributions made by the two companies entering into a transaction. A profit split is then determined based on these contributions. OECD (2014) provides extensive guidelines on pricing transactions involving intangibles.

A.2 OECD Base Erosion and Profit Shifting Project

In what follows we briefly summarize the key provisions of reform proposed by OECD/G20 Inclusive Framework on BEPS, as they were at the time of the writing of this paper.[21]

A.2.1 Pillar 1: Profit allocation and nexus

The general principle behind Pillar 1 is to allocate taxing rights more closely where the customers and users of the in-scope MNEs are located. The key elements of Pillar 1 are as follows.

Scope. The new profit allocation rule will apply to groups with greater than €20 billion in worldwide revenues and a profitability before tax margin of at least 10 percent. There are some exclusions for extractive industries and regulated financial services.

Nexus. The allocation key is based on the revenue that is sourced to each jurisdiction. It will be sourced to the end-market jurisdictions, where goods or services are used or consumed, permitting allocation to a market jurisdiction from which the in-scope MNE derives at least €1 million in revenues.

Quantum. For in-scope MNEs, 25% of residual profit (i.e. profit in excess of 10% of revenue) will be allocated to market jurisdictions with nexus using a revenue-based allocation key.

Elimination of double taxation. Profit allocated to a market jurisdiction will be dispensed from double taxation through direct exemption of credit method.

Unilateral Measures. The agreement requires all parties to remove all digital services taxes and other relevant, similar measures with respect to all companies and to commit not to introduce such measures in the future.

A.2.2 Pillar 2: Global minimum taxation

The second pillar consists of two sets of rules granting jurisdictions additional taxing rights: (i) interlocking domestic rules termed Global anti-Base Erosion (GloBE) rules, and (ii) a treaty-based Subject to Tax Rule (STTR). Their key features are as follows.

Scope. GloBe rules apply to multinational enterprise groups with a total consolidated group revenue above €750 million in at least two of the four preceding years.

Minimum tax rate. GloBE rules apply a system of top-up taxes that brings the total amount of taxes paid on an MNE's profit in a jurisdiction up to the minimum rate of 15%.

Exclusions. GloBe rules will also provide for an exclusion for those jurisdictions where the MNE has revenues of less than EUR 10 million and profits of less than EUR 1 million.

[21] The details of both pillars as well as the exact implementation plan are very much a work in progress at the time of writing this paper. Since November 2021 the OECD has been organizing a series of public consultation meetings in order to work out technical details and parameters of the reform.

Subject to Tax Rule (STTR). This complements the GloBE rules by targeting intra-MNE payments exploiting certain provisions of the treaty to shift profits from source countries to payee jurisdictions where those payments are subject to no or low rates of nominal taxation. In such cases, it reallocates taxing rights to source jurisdictions. It applies to such payments as covered payments—interest, royalties, brokerage, marketing, procurement, agency or other intermediary services, and so on. The minimum rate for the STTR will be 9 percent.

B Data sources

World Development Indicators. Data on population and output come from the World Bank's World Development Indicators database. The specific series that we use are total population (SP.POP.TOTL), GDP in current US dollars (NY.GDP.MKTP.CD), and GDP at purchasing power parity in constant 2011 international dollars (NY.GDP.MKTP.PP.KD). For each of these variables, when constructing regional aggregates, we sum across countries within a region following McGrattan and Waddle (2020), and then average over the period 2014–2017.

World Input-Output Database. International goods trade data are taken from the World Input-Output Database (Timmer et al., 2015). For each bilateral import relationship, we sum all intermediate inputs and final uses of goods (industries 1–23, which represent agriculture, resource extraction, and manufacturing) from countries in the source region by countries in the destination region. We use data from 2014, the most recent year available.

OECD AMNE Database. This is a new dataset provided by the OECD which distinguishes between three types of firms: foreign affiliates (firms with at least 50% foreign ownership), domestic MNEs (domestic firms with foreign affiliates), and domestic firms not involved in international investment. It includes a full matrix of the output of foreign affiliates in 59 countries plus the rest of the world (in the host country, industry, parent country dimension), as well as matrices for value-added and for exports and imports of intermediate inputs (host country and industry). A second set of matrices in the database provides information on output, value-added, and exports and imports of intermediate inputs of domestic MNEs and non-MNE domestic firms (from 2008 onwards). In addition, split Inter-Country Input-Output tables are provided distinguishing for all countries the transactions of domestic-owned and foreign-owned firms. These tables can be used to analyze multinational production in value-added terms. We exploit them to discipline our model and make sure it replicates the share of each region's gross value added that is accounted for by foreign multinationals. We first map the set of 59 countries from the AMNE dataset to our five regions and then compute the average value-added shares for three types of firms (foreign affiliates, domestic MNES, and domestic non-MNEs) in each region over the time period 2008–2016. The data can be accessed at OECD AMNE Database.

Compustat. Data on sales, employment, and country of origin of parent companies come from the Compustat North America Fundamentals Annual database. This database contains data of North American companies parsed from SEC filings. Data on subsidiaries come from the Wharton Research and Data Services (WRDS) Subsidiary Data. These data also come from SEC filing, particularly Exhibit 21, in which firms filing with the SEC must list the names of all existing Significant Subsidiaries. For a detailed, legal definition of Significant Subsidiaries, see here. Roughly, if the parent company controls at least 10% of the subsidiary, it is considered Significant. The WRDS data are available from 1995 to 2019, and contain identifying information for the parent company, as well as the name and country of residence of all Significant Subsidiaries. These two datasets were linked using a common identifier of the parent company, the gvkey. Mean and median sales and employment statistics were computed for the years 2010-2019. The unit of observation was parent company-year.

U.S. Census Data. To discipline the firm size distribution we exploit data from the Statistics of U.S. Businesses (SUSB). SUSB is an annual series that provides national and subnational data on the distribution of economic data by establishment industry and enterprise size. SUSB covers most of the country's economic activity. The series excludes data on nonemployer businesses, private households, railroads, agricultural production, and most government entities. We construct a Lorenz employment curve for the U.S. at the firm level using two Excel spreadsheets available at the Census website. We combine the table with detailed employment sizes with the table with larger employment sizes (20,000+ employees), both from 2019 SUSB. This allows us to account for a long right tail of the firm size distribution in our model, which is crucial given that average MNE is three orders of magnitude larger than the average firm in the U.S. economy. Both Excel files can be downloaded from the SUSB website.

Tørsløv et al. (2022). Two kinds of data are taken from this paper: lost profits and effective corporate income tax rates. Total lost profits are from sheet Table3 of the Main Data Excel file. We first sum across all countries within the North America and Europe regions, and then set the rest of the world's lost profits by subtracting the North America and Europe totals from the overall world total. The share of lost profits that are shifted to the tax haven region is constructed in the same way using sheet TableC2 in the Replication Guide Tables Excel file. The effective corporate income tax rates come from sheet DataF2 in the Main Tables Excel file. Here, we take the average across countries within each region. Both Excel files can be downloaded from `https://missingprofits.world/`.

C Elasticity of profit shifting margin

In this section we briefly discuss the empirical literature on profit shifting, which aims to estimate the elasticity of reported profits with respect to the tax rate differentials across jurisdictions. We begin with an overview of the empirical strategy adopted in this line of research, then move to the discussion of the headline, consensus estimates emerging from the literature. Finally we link our structural modelling approach to the empirical strategy.

C.1 Empirical strategy

Most of the empirical literature on elasticity of the profit shifting margin follows the concept presented by Grubert and Mutti (1991) and Hines and Rice (1994) that the reported pre-tax profit of a multinational entity, Π_i^R, is a sum of the "true" profit, Π_i^T, and the profit shifted for tax reasons, Π_i^S

$$\Pi_i^R = \Pi_i^T + \Pi_i^S. \tag{68}$$

This shifted profit would be positive in low-tax countries and negative in high-tax countries. The idea here is that the actual profitability of multinational enterprises with similar characteristics (e.g. size, industry, country etc.) is similar. However, the opportunities to shift profits differ since they depend on such characteristics as locations of the other subsidiaries and statutory tax rates in these locations. Thus, the entities linked to low-tax jurisdictions are more likely to shift profits and the entities linked to high-tax jurisdictions are more likely to receive profits. The fundamental challenge for estimating the elasticity of profit shifting margin is that neither "true" profits nor shifted ones are directly observable in the firm-level data. To tackle this problem the literature usually assumes that "true" profits are equal to output minus the wage bill, with the wage being equal to marginal product of labor (see for example Huizinga and Laeven (2008)). As for the shifted profits, the literature typically specifies some stylized framework that allows linking shifted profits to tax differentials between jurisdiction j and other operating jurisdictions. This strategy leads to the following generic equation to identify shifting profits:

$$\pi^R_{i,j,t} = \beta X_{i,j,t} - \gamma C_{i,j,t} + \delta_t + \varepsilon \tag{69}$$

where $\pi^R_{i,j,t} = \ln \Pi^R_{i,j,t}$ are log reported profits of a multinational i located in jurisdiction j at time t, $X_{i,j,t}$ is a vector of determinants of true profitability, which includes capital and labor inputs among others. It may also include a number of macroeconomic variables, such as GDP growth, exchange rate, or inflation. $C_{i,j,t}$ is a composite variable that summarizes the tax differentials between jurisdiction j and other jurisdictions in which the MNE located in jurisdiction i has subsidiaries. The specific formula for $C_{i,j,t}$ differs across papers but in all of them it reflects the tax incentives to shift profits away from or into jurisdiction j. Finally δ_t denotes time fixed-effect and ε denotes the residual term. The coefficient of interest is then γ which reflects the extent to which the multinational shifts profits into or out of affiliate i. It is important to note that this estimate represents a marginal effect – i.e. the change in reported profits associated with a small change in tax rates, *holding all else constant*. We can interpret γ in equation (69) as the semi-elasticity of observed profits π^O_i with respect to the composite tax variable $C_{i,j,t}$. The *semi-elasticity* indicates the percentage change of reported profit in response to a one percentage point change in the tax differential vis-a-vis other international locations, reflecting the incentive to shift profits abroad:

$$\text{Semi-Elasticity} = \frac{\partial \ \ln \text{ Reported Profits}}{\partial \ \text{Tax Differential}} \approx \frac{\partial \ \text{Reported Profit}}{\text{Reported Profit}} \times \frac{1}{\partial \ \text{Tax Differential}}.$$

Note that differentiating (69) we get

$$\frac{\partial \pi^R_{i,j,t}}{\partial C_{i,j,t}} = -\frac{\partial \Pi^R_{i,j,t}}{\Pi^R_{i,j,t}} \cdot \frac{1}{\partial C_{i,j,t}} = \gamma, \tag{70}$$

thus γ reflects the semi-elasticity of interest.

C.2 Empirical estimates

A number of papers estimate different versions of equation (69) for a variety of datasets and time periods. A thorough and detailed review of this literature is beyond the scope of this paper.[22] Instead, we focus here on the two most recent survey papers, which conduct meta analyses of existing estimates, and on the main OECD estimate, all of which report the headline semi-elasticity number.

[22]See Dharmapala (2014), Heckemeyer and Overesch (2017), Johansson et al. (2017) and Beer et al. (2020) for extensive reviews of this line of research.

Johansson et al. (2017) provide the main estimate of the magnitude of the profit shifting used by the OECD. They conduct a comprehensive study using firm-level data from the ORBIS database to assess international tax planning by multinational enterprises (MNEs). Their results are based on an impressively large sample of firms (1.2 million observations of MNE accounts) in 46 OECD and G20 countries and a sophisticated procedure to identify MNE groups. Their headline estimate of the semi-elasticity of the profit shifting margin with respect to the tax differential is **1.11** (see Table 1, column 1 and footnote 31 in their paper). Hence, reported profits decrease by about 1.1% if the international tax rate differential increases by one percentage point. The estimated elasticities combined with a number of assumptions are then used to estimate the effect of international tax planning on corporate tax revenues: the estimated net tax revenue loss ranges from 4% to 10% of global corporate tax revenues.

Heckemeyer and Overesch (2017) construct a meta-database containing 203 primary estimates sampled from 27 empirical studies identified by means of article search engines. All of the included studies estimate the empirical relationship between reported parent and subsidiary profitability and the tax incentive to shift profits abroad. Therefore, this meta-analysis reviews the literature, providing indirect evidence for profit shifting without specifying directly the shifting methods. They find a tax semi-elasticity of pre-tax profit of about **0.79**, in absolute terms. They conclude that across all specifications the predicted semi-elasticities turn out to be statistically significant and rather robust in magnitude. They also provide a 95% confidence interval in addition to the point estimate and conclude that conditional on a hypothetical state-of-the-art study design, the set of semi-elasticities that should not be rejected at the 5% significance level ranges from 0.546 to 1.026.

Beer, de Mooij, and Liu (2020) extend the analysis conducted by Heckemeyer and Overesch (2017) and include 11 additional studies and 199 additional primary estimates. They also reduce specification bias, and adopt an enhanced estimation method that corrects for within-study correlation of primary estimates. Their results indicate that a semielasticity of reported pretax profits with respect to international tax differentials equal to **0.98** is a good reflection of the literature. This means that a one-percentage-point larger tax rate differential reduces reported pretax profits of an affiliate by 1%.

C.3 Model counterpart of semi-elasticity

We now describe how we estimate the model counterpart of the semi-elasticity summarized above. We view this as a validation exercise of the cost function $C(\lambda)$ upon which the extent of profit shifting in the presence of tax differentials between jurisdictions heavily depends. Since our parsimonious model of only four productive regions does not provide sufficient variation in cross-jurisdiction differences in corporate tax rates (regressor $C_{i,j,t}$ in equation (69)), we conduct a simulation exercise as follows.

We simulate 100 counterfactual economies, raising the corporate tax rate of the LT region incrementally for the first 50 economies and the rate of the TH region for the latter 50. We set the highest counterfactual corporate tax rate to 15%, equal to the global minimum tax rate of OECD Pillar 2. In each of these counterfactual economies, we hold fixed the set of firms' FDI and exporting destinations, J_F and J_X, as well as the final good price and wage rate of each region, P_i and W_i. We allow firms to solve for their optimal choices of labor $\boldsymbol{\ell}$, intangible capital z and shifting shares λ_{LT} and λ_{TH}. In other words, the firms' problem is re-solved in a partial equilibrium setting, which allows us to isolate the relationship of reported profits in home divisions to tax rate differentials relative to the profit-shifting destination.

Denote k as the index of a counterfactual economy. We follow the empirical specification of equation

(69) and run the regression using the model-simulated dataset:

$$\log \pi_i^{k,PS}(\omega) = \beta_0 + \beta_\ell \log \ell_i^k(\omega) + \beta_z \log z^k(\omega) - \beta_\tau \hat{\tau}_i^k + \epsilon_i^k(\omega) \tag{71}$$

where we denote by τ_i^k the counterfactual tax differential defined as $\hat{\tau}_i^k = \tau_i - \tau_{LT}^k$ for $k \leq 50$ and $\hat{\tau}_i^k = \tau_i - \tau_{TH}^k$ otherwise. For each experiment k, we include in the regression only home divisions of firms doing FDI in the region for which we change the corporate tax rate. We only include home divisions of profit-shifting MNEs because we do not model profit shifting originating from a foreign subsidiary. Nonetheless, such regression informs us of how reported profit responds to changes in profit-shifting relevant tax differentials, which is captured by the coefficient of interest β_τ. We report the coefficient estimate of β_τ in Table 3.

D Sensitivity Analysis

Our quantitative results are robust to a variety of alternative assumptions and calibrations. Here, we describe the results of three sensitivity analyses that illustrate the impact of some of the most important elements of our model and policy experiments. First, we analyze an alternative profit-reallocation rule for OECD BEPS Pillar 1 based on output shares instead of sales shares. Second, we analyze the role of the intangible capital income share. Last, we explore the sensitivity of our results to the costs of shifting profits. Table 6 shows the results of these sensitivity analyses.

Alternative profit reallocation rules. The first pillar of the OECD's BEPS project reallocates the rights to tax a portion of a firm's global profits to the regions in which it operates in accordance with these regions' shares of the firm's global sales. Importantly, some of these rights are allocated to a firm's export markets, even if the firm does not operate foreign affiliates in these markets. This aspect of the rule increases effective tax rates for firms based in Europe, the low-tax region, and the rest of the world because North America, which is a large, rich export market, has the highest corporate income tax rate. This reduces these firms' incentives to invest in intangible capital, even if they do not shift profits at all. This partly explains why Pillar 1 has larger macroeoconomic consequences than Pillar 2, despite having smaller effects on profit shifting. To explore the importance of this aspect of Pillar 1, we have analyzed the effects of alternative versions in which profit taxation rights are allocated for MNEs only, or are based on output shares instead of sales shares. Panel (a) of Table 6 shows the effects of a profit allocation rule that applies only to MNEs, as opposed to firms that export but do not operate foreign affiliates. The effects on profit shifting are the same as the OECD's version but the macroeconomic consequences are smaller, especially outside of North America. Panel (b) of Table 6 shows what happens when profit-taxation rights are allocated based on output rather than sales. Under this version of the pillar, export destinations do not receive any taxation rights at all. The results are almost identical to panel (a). These results indicate that allocating taxation rights based on export sales should be avoided.

Intangible share. We have set the share of intangible capital in production, ϕ, to match the share of income that accrues to intangible capital in MNEs' foreign affiliates. This approach ensures that our model captures the extent to which nonrivalry governs MNEs' incentives to invest in intangible capital. This share is the key determinant of the potential scope for profit shifting; a greater intangible share means more licensing fee income that can be transferred to the low-tax region and/or the tax haven. Of course, it is also the key determinant of the macroeconomic impact of policies that affect incentives to invest in intangible capital, including the policies designed to reduce profit shifting that we study. Panels (c) and (d) of Table 6 show the results of our experiments under alternative calibrations with different intangible shares. In each, we recalibrate all model parameters except for those that govern profit shifting. This allows us to explore how the intangible share affects profit shifting under the current international tax system as well as the effects of changes to this system. The results of these analyses show that a lower intangible share reduces macroeconomic effects of transfer pricing and profit shifting, reduces the amount of profit shifting under the current tax code, and reduces the macroeconomic consequences of the OECD BEPS pillars; the reverse is true for a higher intangible share. However, the extent to which the BEPS pillars reduce profit shifting is about the same as in the baseline model. For example, with a lower intangible share, lost profits in North America fall by 1 - 0.27/0.45 = 40% under Pillar 1, exactly the same as in the baseline model.

Labor supply. In our baseline model we assume that preferences are log-separable in consumption and leisure as in McGrattan and Waddle (2020). This means that labor supply is driven partly by income effects. This is particularly important for the low-tax region, where lump-sum transfers of taxes on profits shifted away from high-tax countries are fairly sizeable. Here, we explore alternative versions of the model without income effects on labor supply. Panel (e) of Table 7 shows the results of when preferences are assumed to be GHH, and panel (f) shows the results from an even starker assumption of perfectly inelastic labor supply. Both versions yield similar results. For the three high-tax regions, the results are broadly similar to the baseline, with all variables moving in the same direction in all experiments. In the low-tax region, though, profit shifting now raised GDP instead of lowering it, and the OECD/G20 BEPS pillars reduce GDP much more than in the baseline. This highlights a somewhat important point for the low-tax country: profit shifting causes something akin to Dutch disease (which is kind of funny, since the Netherlands is in our low-tax region).

Profit-shifting costs. We have set the costs of profit shifting, ψ_{iLT} and ψ_{iTH}, to match estimates in the literature about the amount of profit shifting and the extent to which profits are shifted to low-tax "productive" regions versus "unproductive" tax havens. These estimates are inferred from information about the profitability and labor shares of MNEs' foreign affiliates in these regions—it is impossible to measure lost profits directly without access to detailed information about intra-MNE transactions—so there is some uncertainty about how much profit shifting truly occurs. To determine the sensitivity of our results to these key parameters, we have conducted our experiments in alternative calibrations within which these parameters are set to higher or lower values. Panel (g) of Table 6 shows the results when ψ_{iLT} and ψ_{iTH} are halved, while panel (h) shows the results when they are doubled. With lower profit-shifting costs, there is more profit shifting under the current tax system and the OECD BEPS pillars have larger macroeconomic effects; the reverse holds with lower costs. As in the previous exercise, the BEPS pillars reduce profit shifting by about the same amount as in the baseline. For example, with lower shifting costs, lost profits in North America fall by 1 - 1.26/2.03 = 38% under Pillar 1.

Table 6: Sensitivity analysis

Experiment	Lost profits (benchmark = 1) North America	Europe	Low tax	Rest of World	GDP (% chg.) North America	Europe	Low tax	Rest of World
(a) Profit reallocation rule applies to MNEs only								
Pillar 1	0.60	0.66	0.69	0.63	-0.12	-0.10	-0.06	-0.09
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.16	-0.12	-0.06	-0.10
(b) Production-based profit reallocation rule								
Pillar 1	0.60	0.66	0.69	0.63	-0.12	-0.09	-0.06	-0.09
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.16	-0.11	-0.06	-0.09
(c) Low intangible share								
Effects of transfer pricing	–	–	–	–	-0.06	-0.08	-0.12	-0.08
Effects of profit shifting	0.45	0.45	0.48	0.44	0.02	0.02	-0.04	0.01
Pillar 1	0.27	0.30	0.33	0.28	-0.08	-0.11	-0.12	-0.10
Pillar 2	0.17	0.12	0.23	0.07	-0.02	-0.01	0.02	0.00
Pillars 1 & 2 together	0.10	0.08	0.16	0.04	-0.10	-0.12	-0.13	-0.10
(d) High intangible share								
Effects of transfer pricing	–	–	–	–	-0.27	-0.26	-0.38	-0.28
Effects of profit shifting	1.60	1.60	1.60	1.63	0.16	0.12	-0.04	0.09
Pillar 1	0.96	1.06	1.10	1.03	-0.19	-0.18	-0.14	-0.17
Pillar 2	0.59	0.42	0.78	0.25	-0.11	-0.05	0.02	-0.03
Pillars 1 & 2 together	0.36	0.28	0.53	0.16	-0.25	-0.22	-0.15	-0.18

Notes: In panel (a), the profit-reallocation rule for pillar 1 applies only to MNEs (not firms that export but do not operate foreign affiliates). In panel (b), the rule is based on value added rather than sales; profits are not reallocated to export destinations. In panels (c) and (d), the intangible share is changed and all parameters except for profit-shifting costs are recalibrated. Lost profits are measured relative to the benchmark equilibrium in the baseline calibration.

Table 7: Sensitivity analysis, continued

Experiment	Lost profits (benchmark = 1) North America	Europe	Low tax	Rest of World	GDP (% chg.) North America	Europe	Low tax	Rest of World
(e) GHH preferences								
Effects of transfer pricing	–	–	–	–	-0.11	-0.21	-0.20	-0.18
Effects of profit shifting	1.00	1.00	1.00	1.00	0.01	-0.03	0.31	-0.01
Pillar 1	0.60	0.66	0.69	0.63	-0.09	-0.10	-0.35	-0.10
Pillar 2	0.37	0.26	0.49	0.15	0.00	0.06	-0.16	0.03
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.09	-0.07	-0.46	-0.09
(f) Fixed labor supply								
Effects of transfer pricing	–	–	–	–	-0.09	-0.21	-0.21	-0.18
Effects of profit shifting	1.00	1.00	1.00	1.00	-0.02	-0.05	0.24	-0.02
Pillar 1	0.60	0.66	0.69	0.63	-0.06	-0.09	-0.32	-0.10
Pillar 2	0.37	0.26	0.49	0.15	0.02	0.07	-0.11	0.04
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.06	-0.06	-0.38	-0.08
(g) Low profit-shifting costs								
Effects of profit shifting	2.03	1.96	1.90	2.05	0.15	0.10	-0.09	0.06
Pillar 1	1.26	1.31	1.33	1.32	-0.16	-0.16	-0.10	-0.14
Pillar 2	0.79	0.53	0.96	0.33	-0.10	-0.03	0.06	-0.01
Pillars 1 & 2 together	0.48	0.35	0.66	0.21	-0.23	-0.19	-0.09	-0.15
(h) High profit-shifting costs								
Effects of profit shifting	0.48	0.50	0.51	0.47	0.04	0.04	-0.02	0.03
Pillar 1	0.28	0.32	0.35	0.29	-0.12	-0.13	-0.14	-0.13
Pillar 2	0.17	0.13	0.25	0.07	-0.03	-0.01	0.01	-0.00
Pillars 1 & 2 together	0.10	0.09	0.17	0.04	-0.14	-0.15	-0.15	-0.13

Notes: In panel (e), households have GHH preferences (no income effects on labor supply). In panel (f), labor supply is fixed. In panel (g), the parameters that govern the marginal cost of profit shifting, ψ_{ij}, are halved; in panel (h), they are doubled.

E Proofs of Analytical Results

This Appendix contains the proofs of the lemmas and propositions from the main body of the paper.

E.1 Main Lemmas

Proof of Lemma 1.

Rewrite the problem (10) using definitions of profits as

$$\begin{aligned}\max_{z,\lambda,\{\ell_i\}_{i=1}^{I}} &(1-\tau_i)\left(p_i\left(A_i\left(N_i z\right)^{\phi}\ell_i^{\gamma}\right)-w_i\ell_i-p_i z+z\left[\varphi\lambda\sum_k\vartheta_k\left(z\right)-\lambda\vartheta_i\left(z\right)+\left(1-\lambda\right)\sum_{k\neq i}\vartheta_k\left(z\right)-\sum_k\vartheta_k\left(z\right)\mathcal{C}\left(\lambda\right)\right]\right)\\
&+\left(1-\tau_{i^*}\right)\left(p_{i^*}\left(A_{i^*}\left(N_{i^*}z\right)^{\phi}\ell_{i^*}^{\gamma}\right)-w_{i^*}\ell_{i^*}+z\left[\lambda\sum_{k\neq i^*}\vartheta_k\left(z\right)-\left(1-\lambda\right)\vartheta_{i^*}\left(z\right)-\varphi\lambda\sum_k\vartheta_k\left(z\right)\right]\right)\\
&+\left(1-\tau_k\right)\sum_{k\neq i,i^*}\left(p_k\left(A_k\left(N_k z\right)^{\phi}\ell_k^{\gamma}\right)-w_k\ell_k-\vartheta_k\left(z\right)z\right). \qquad (72)\end{aligned}$$

The FOCs are then:

$$\ell_i: 0=\gamma p_i A_i\left(N_i z\right)^{\phi}\ell_i^{\gamma-1}-w_i,\ \ i=1,2,...,I, \qquad (73)$$

$$\begin{aligned}z: 0=&\sum_k\left(1-\tau_k\right)\phi N_k p_k A_k\left(N_k z\right)^{\phi-1}\ell_k^{\gamma}+\left(1-\tau_i\right)\left[-p_i+\varphi\lambda\sum_k\vartheta_k\left(z\right)-\lambda\vartheta_i\left(z\right)+\left(1-\lambda\right)\sum_{k\neq i}\vartheta_k\left(z\right)-\sum_k\vartheta_k\left(z\right)\mathcal{C}\left(\lambda\right)\right]\\
&+\left(1-\tau_{i^*}\right)\left[\lambda\sum_{k\neq i^*}\vartheta_k\left(z\right)-\left(1-\lambda\right)\vartheta_{i^*}\left(z\right)-\varphi\lambda\sum_k\vartheta_k\left(z\right)\right]-\sum_{k\neq i,i^*}\left(1-\tau_k\right)\vartheta_k\left(z\right), \qquad (74)\end{aligned}$$

$$\begin{aligned}\lambda: 0=&\left(1-\tau_i\right)z\left[\varphi\sum_k\vartheta_k\left(z\right)-\vartheta_i\left(z\right)-\sum_{k\neq i}\vartheta_k\left(z\right)-\sum_k\vartheta_k\left(z\right)\mathcal{C}'\left(\lambda\right)\right] \qquad (75)\\
&+\left(1-\tau_{i^*}\right)z\left[\sum_{k\neq i^*}\vartheta_k\left(z\right)+\vartheta_{i^*}\left(z\right)-\varphi\sum_k\vartheta_k\left(z\right)\right].\end{aligned}$$

Inspect the FOC wrt to λ:

$$\begin{aligned}0&=\left(1-\tau_i\right)z\left[\varphi\sum_k\vartheta_k\left(z\right)-\vartheta_i\left(z\right)-\sum_{k\neq i}\vartheta_k\left(z\right)-\sum_k\vartheta_k\left(z\right)\mathcal{C}'\left(\lambda\right)\right]+\left(1-\tau_{i^*}\right)z\left[\sum_{k\neq i^*}\vartheta_k\left(z\right)+\vartheta_{i^*}\left(z\right)-\varphi\sum_k\vartheta_k\left(z\right)\right]\\
0&=\left(1-\varphi\right)\sum_k\vartheta_k\left(z\right)\left(\tau_i-\tau_{i^*}\right)-\left(1-\tau_i\right)\sum_k\vartheta_k\left(z\right)\mathcal{C}'\left(\lambda\right),\end{aligned}$$

which yields

$$\lambda=\left(\mathcal{C}'\right)^{-1}\left[\left(1-\varphi\right)\frac{\left(\tau_i-\tau_{i^*}\right)}{1-\tau_i}\right]. \qquad (76)$$

Under Assumption 1 this can be written as

$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right).$$

Now towards proving the lemma, we have

$$\frac{\partial \lambda}{\partial \varphi} = -\exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right)\left(\frac{\tau_i - \tau_{i^*}}{1-\tau_i}\right) < 0,$$
$$\frac{\partial \lambda}{\partial \tau_{i^*}} = -\exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right)\left(\frac{1-\varphi}{1-\tau_i}\right) < 0,$$

and therefore the elasticities are

$$\varepsilon^{\lambda}_{\varphi} = \frac{\partial \lambda}{\partial \varphi}\frac{\varphi}{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)} = -\exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right)\left(\frac{\tau_i - \tau_{i^*}}{1-\tau_i}\right)\frac{\varphi}{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}$$
$$= -\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{\tau_i - \tau_{i^*}}{1-\tau_i}\right)\varphi,$$

$$\varepsilon^{\lambda}_{\tau_{i^*}} = \frac{\partial \lambda}{\partial \tau_{i^*}}\frac{\tau_{i^*}}{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)} = -\exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right)\left(\frac{1-\varphi}{1-\tau_i}\right)\frac{\tau_{i^*}}{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}$$
$$= -\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{1-\varphi}{1-\tau_i}\right)\tau_i^*,$$

which proves 1. and 2. ■

The following lemma will be useful in our further derivations.

Lemma 2 *The allocations of intangible capital are as follows:*

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{77}$$
$$z^{TP} = \left(\frac{\sum_k \Lambda_k}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{78}$$
$$z^{PS} = z^{TP}\left((1-\mathcal{C}(\lambda)) + \frac{\lambda(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}. \tag{79}$$

Proof. Free transfer of z requires $\vartheta_k(z) = 0$ thus the (74) becomes

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}},$$

and hence we obtain (77). For the transfer pricing case we have $\lambda = 0$ and the (74) becomes

$$0 = z^{\frac{\phi+\gamma-1}{1-\gamma}}\sum_k (1-\tau_k)\Lambda_k - (1-\tau_i)p_i - \sum_k \vartheta_k(z)(\tau_i - \tau_k),$$

where

$$\begin{aligned}\vartheta_k(z) &= \phi p_k N_k \left(A_k (N_k z)^{\phi-1} \ell_k^{\gamma}\right) \\ &= \phi p_k N_k \left(A_k (N_k z)^{\phi-1} \left(\frac{\gamma p_i A_i (N_i z)^{\phi}}{w_i}\right)^{\frac{\gamma}{1-\gamma}}\right) \\ &= \phi \gamma^{\frac{\gamma}{1-\gamma}} p_k^{\frac{1}{1-\gamma}} A_k^{\frac{1}{1-\gamma}} \left(\frac{1}{w_k}\right)^{\frac{\gamma}{1-\gamma}} N_k^{\frac{\phi}{1-\gamma}} (z)^{\frac{\phi+\gamma-1}{1-\gamma}} = \Lambda_k (z)^{\frac{\phi+\gamma-1}{1-\gamma}}.\end{aligned}$$

Thus, we have

$$\begin{aligned}0 &= z^{\frac{\phi+\gamma-1}{1-\gamma}} \sum_k (1-\tau_k) \Lambda_k - (1-\tau_i) p_i - \sum_k \Lambda_k (z)^{\frac{\phi+\gamma-1}{1-\gamma}} (\tau_i - \tau_k) \\ z^{TP} &= \left(\frac{\sum_k (1-\tau_k) \Lambda_k}{(1-\tau_i) p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}.\end{aligned}$$

Hence we obtain (78). Now, for the profit shifting case, we can rewrite (74) as

$$\begin{aligned}0 &= z^{\frac{\phi+\gamma-1}{1-\gamma}} \sum_k (1-\tau_k) \Lambda_k - (1-\tau_i) p_i - \sum_k \vartheta_k(z) (\tau_i - \tau_k) - \sum_k \vartheta_k(z) (1-\tau_i) \mathcal{C}(\lambda) + \lambda \sum_k \vartheta_k(z) \left[(1-\varphi)(\tau_i - \tau_{i^*})\right] \\ &= z^{\frac{\phi+\gamma-1}{1-\gamma}} \sum_k (1-\tau_k) \Lambda_k - (1-\tau_i) p_i - z^{\frac{\phi+\gamma-1}{1-\gamma}} \sum_k \Lambda_k \left[(\tau_i - \tau_k) + \lambda (1-\varphi)(\tau_i - \tau_{i^*}) - (1-\tau_i) \mathcal{C}(\lambda)\right],\end{aligned}$$

and thus

$$\begin{aligned}z^{PS} &= \left(\frac{\sum_k \Lambda_k (1-\tau_i) \left[(1-\mathcal{C}(\lambda)) + \frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right]}{(1-\tau_i) p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}} \\ &= z^{TP} \left((1-\mathcal{C}(\lambda)) + \frac{\lambda(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}},\end{aligned}$$

thus we have (79). ■

Now, we move towards proving Lemma 1.

Proof of Proposition 1. Note we have derived the formulas for $z^{FT}, z^{TP}$and z^{PS} and we have the following formulas for λ and $\mathcal{C}(\lambda)$:

$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right), \tag{80}$$

$$\mathcal{C}(\lambda) = \lambda - (\lambda - 1)\log(1-\lambda), \tag{81}$$

where

$$\tau_{i^*} \equiv \min\{\tau_1, ..., \tau_K\}.$$

Start with showing 1. Let

$$\tau_i \equiv \max\{\tau_1, ..., \tau_K\},$$

then

$$1-\tau_i < 1-\tau_k \quad \forall k,$$
$$\frac{1-\tau_i}{1-\tau_i} < \frac{1-\tau_k}{1-\tau_i} \quad \forall k.$$

Thus

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)\,p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}} = \left(\frac{1}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}} \left(\sum_k \frac{(1-\tau_k)}{(1-\tau_i)}\Lambda_k\right)^{\frac{1-\gamma}{1-\phi-\gamma}}$$
$$> \left(\frac{1}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}} \left(\sum_k \frac{(1-\tau_i)}{(1-\tau_i)}\Lambda_k\right)^{\frac{1-\gamma}{1-\phi-\gamma}} = z^{TP},$$

thus we have

$$z^{TP} < z^{FT}.$$

Now, towards showing 2. Start with ($\Leftarrow$) direction, and let $0<\varphi<1$. Then, by (80) we have $0<\lambda<1$. Take any $\lambda \in (0,1)$ and notice that $z^{PS} > z^{TP}$ iff

$$\mathcal{C}(\lambda) < \frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}. \tag{82}$$

Note that $\forall x>0$ we have

$$x < -\ln(1-x)$$
$$\exp(-x) > -x+1$$
$$\left(\frac{1}{1-\exp(-x)}\right) > \frac{1}{x}$$
$$\left(1+\frac{\exp(-x)}{1-\exp(-x)}\right) > \frac{1}{x}.$$

Now, set

$$x \equiv \frac{(1-\varphi)(\tau_i-\tau_{i^*})}{(1-\tau_i)},$$

which implies

$$\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{(1-\tau_i)}\left[1+\frac{\left(\exp\left(-\frac{a(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)\right)}{1-\exp\left(-\frac{a(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}\right] > 1$$
$$\frac{\left(-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)\right)}{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right) > 1-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{(1-\tau_i)}$$
$$\frac{\left(1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)-1\right)}{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}\log\left(1-1+\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)\right) > 1-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{(1-\tau_i)},$$

which using (80) can be written as

$$\frac{(\lambda-1)}{\lambda}\log(1-\lambda)>1-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{(1-\tau_i)}>0,$$

which through the series of iff inequalities can be transformed as follows

$$\begin{aligned}1-\frac{(\lambda-1)}{\lambda}\log(1-\lambda)&<\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\\ \frac{\lambda-(\lambda-1)\log(1-\lambda)}{\lambda}&<\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\\ \mathcal{C}(\lambda)&<\frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}.\end{aligned}$$

This proves (82) and hence establishes $z^{PS}>z^{TP}$. Given that all the inequalities are iffs the reverse argument holds immediately. To show 3. and 4. notice from (80) first, that

$$\frac{\partial\lambda}{\partial\varphi}<0.$$

Now, we want to show

$$\begin{aligned}\frac{\partial z^{PS}}{\partial\varphi}=&z^{TP}\cdot\frac{1-\gamma}{1-\phi-\gamma}\cdot\left((1-\mathcal{C}(\lambda))+\frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}-1}\times\\ &\left(\left(-\mathcal{C}'(\lambda)\frac{\partial\lambda}{\partial\varphi}\right)+\frac{\partial\lambda}{\partial\varphi}\left[\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right]-\lambda\frac{(\tau_i-\tau_{i^*})}{1-\tau_i}\right)\\ &=\frac{1-\gamma}{1-\phi-\gamma}\cdot z^{PS}\left((1-\mathcal{C}(\lambda))+\frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)^{-1}\left(\frac{\partial\lambda}{\partial\varphi}\left[\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}-\mathcal{C}'(\lambda)\right]-\lambda\frac{(\tau_i-\tau_{i^*})}{1-\tau_i}\right)<0.\end{aligned}$$

This is negative if

$$\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}-\mathcal{C}'(\lambda)\leq 0,$$

and it holds with equality, since it is the condition equalizing marginal cost with marginal benefit of profit shifting λ. Thus, we get

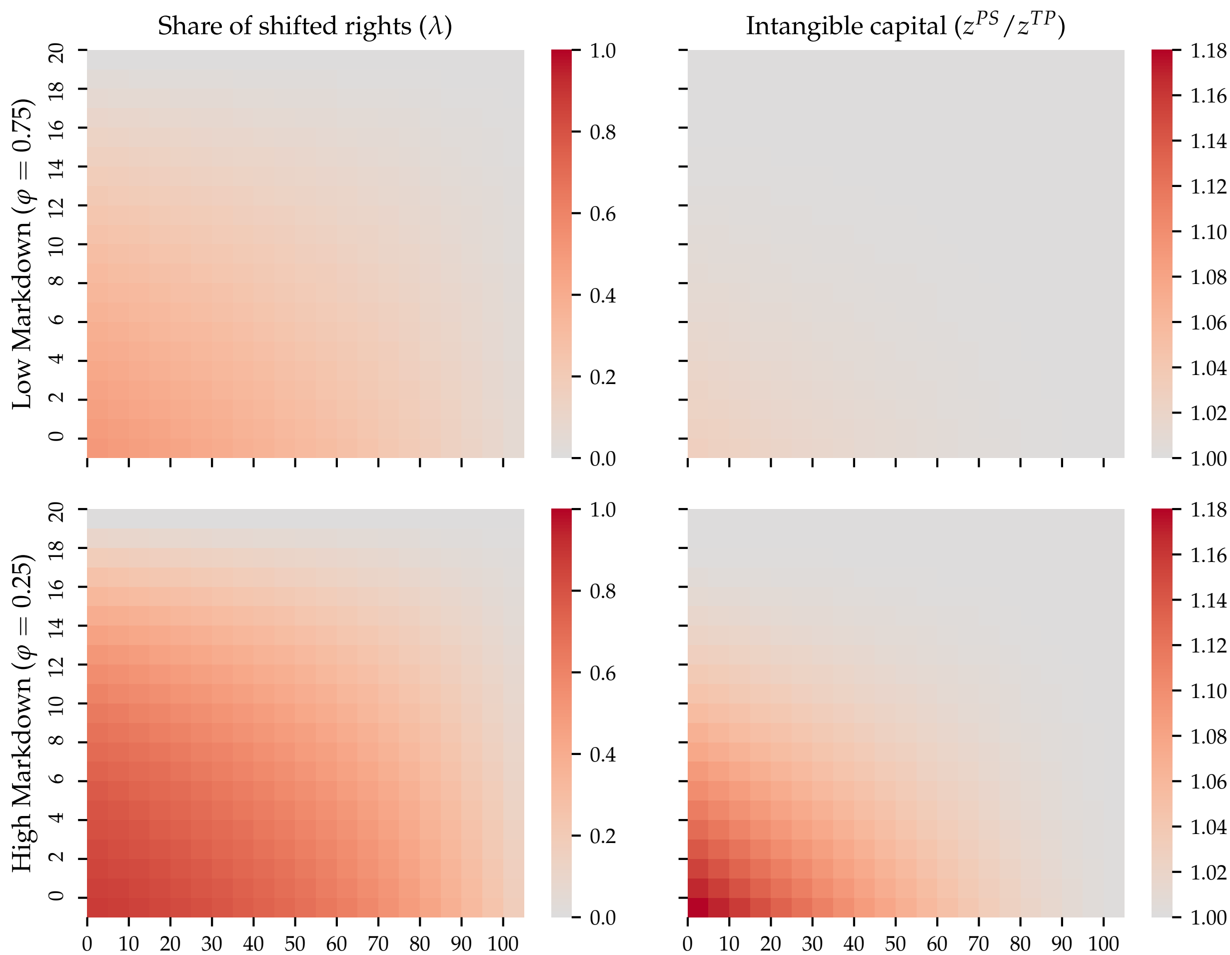
$$\frac{\partial z^{PS}}{\partial\varphi}=\frac{1-\gamma}{1-\phi-\gamma}\cdot z^{PS}\left((1-\mathcal{C}(\lambda))+\frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)^{-1}\left(-\lambda\frac{(\tau_i-\tau_{i^*})}{1-\tau_i}\right)<0,$$

which proves 3. Notice, that proof for 4. follows analogously. Now towards deriving the elasticity

$$\begin{aligned}\varepsilon^{z}_{\tau_{i^*}}&=\frac{1-\gamma}{1-\phi-\gamma}\left((1-\mathcal{C}(\lambda))+\frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)^{-1}\left(-\frac{\tau_{i^*}\lambda(1-\varphi)}{1-\tau_i}\right)\\ &=\frac{1-\gamma}{1-\phi-\gamma}\frac{-\tau_{i^*}\lambda(1-\varphi)}{\lambda(1-\varphi)(\tau_i-\tau_{i^*})\left[1+\frac{(1-\tau_i)(1-\mathcal{C}(\lambda))}{(\tau_i-\tau_{i^*})\lambda(1-\varphi)}\right]}\\ &=\frac{1-\gamma}{1-\phi-\gamma}\left(\frac{-\tau_{i^*}}{\tau_i-\tau_{i^*}}\right)\frac{1}{\left[1+\frac{1-\mathcal{C}(\lambda)}{\lambda\mathcal{C}'(\lambda)}\right]}<0.\end{aligned}$$

■

Figure 2: Comparative statics: Shifted property rights and intangible capital



Notes: X-axis in each plot represents the reallocation share for Pillar 1. Y-axis in each plot represents the global minimum corporate income tax rate for Pillar 2. The comparative statics is computed using the corporate income taxes, TFP and populations as in the quantitative model. All prices are normalized to 1. We set $\phi = 0.11$ and $\varphi = 0.64$. The results are presented for North America.

E.1.1 Proofs under Alternative Assumption

Here, we assume alternatively that MNEs internalize the effect of changing z on the licensing fee $\vartheta_k(z)$ and solve for optimal z under different scenarios (FT, TP, and PS). We then prove Proposition 1 under this assumption. Note that the optimal shifting share λ will not be changed as it is solved independently from z so Lemma 1 holds automatically. Let's first solve for optimal z under this assumption.

Lemma 3 *The allocations of intangible capital are as follows:*

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)\, p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{83}$$

$$z^{TP} = \left[\frac{\sum_k (1-\tau_k)\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (\tau_i-\tau_k)\Lambda_k}{(1-\tau_i)\, p_i}\right]^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{84}$$

$$z^{PS} = \left[\frac{-\frac{\phi}{1-\gamma}\mathcal{C}(\lambda)\sum_k \Lambda_k}{p_i} + \frac{\sum_k (1-\tau_k)\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (\tau_i-\tau_k)\Lambda_k + \lambda\frac{\phi}{1-\gamma}(\tau_i-\tau_{i^*})(1-\varphi)\sum_k \Lambda_k}{(1-\tau_i)\, p_i}\right]^{\frac{1-\gamma}{1-\phi-\gamma}}. \tag{85}$$

Proof of Lemma 3. Starting from the profit maximization problem of an MNE:

$$\begin{aligned}
\max_{z,\lambda,\{\ell_i\}_{i=1}^{I}} (1-\tau_i) & \left(p_i\left(A_i (N_i z)^{\phi}\ell_i^{\gamma}\right) - w_i\ell_i - p_i z + z\left[\varphi\lambda\sum_k \vartheta_k(z) - \lambda\vartheta_i(z) + (1-\lambda)\sum_{k\neq i}\vartheta_k(z) - \sum_k \vartheta_k(z)\,\mathcal{C}(\lambda)\right]\right) \\
&+ (1-\tau_{i^*})\left(p_{i^*}\left(A_{i^*}(N_{i^*}z)^{\phi}\ell_{i^*}^{\gamma}\right) - w_k\ell_{i^*} + z\left[\lambda\sum_{k\neq i^*}\vartheta_k(z) - (1-\lambda)\vartheta_{i^*}(z) - \varphi\lambda\sum_k\vartheta_k(z)\right]\right) \\
&+ (1-\tau_k)\sum_{k\neq i,i^*}\left(p_k\left(A_k(N_k z)^{\phi}\ell_k^{\gamma}\right) - w_k\ell_k - \vartheta_k(z)\, z\right).
\end{aligned} \tag{86}$$

With the derivative of $\vartheta_k(z)$ with respect to z taken, the FOC with respect to z is then:

$$\begin{aligned}
0 = & \sum_k (1-\tau_k)\,\phi N_k p_k A_k (N_k z)^{\phi-1}\ell_k^{\gamma} - (1-\tau_i)\, p_i \\
& - \left[\sum_{k\neq i,i^*}(1-\tau_k)\vartheta_k(z) - \sum (1-\tau_i)\vartheta_k(z) + (1-\tau_{i^*})\vartheta_{i^*}(z)\right] - (1-\tau_i)\sum_k\vartheta_k(z)\,\mathcal{C}(\lambda) \\
& - z\left[\sum_{k\neq i,i^*}(1-\tau_k)\vartheta_k'(z) - \sum_{k\neq i}(1-\tau_i)\vartheta_k'(z) + (1-\tau_{i^*})\vartheta_{i^*}'(z)\right] - (1-\tau_i)\, z\sum_k\vartheta_k'(z)\,\mathcal{C}(\lambda) \\
& + \lambda\Bigg[(1-\tau_i)\varphi\sum_k\vartheta_k(z) - (1-\tau_i)\vartheta_i(z) - (1-\tau_i)\sum_{k\neq i}\vartheta_k(z) + (1-\tau_{i^*})\sum_{k\neq i^*}\vartheta_k(z) + \\
& \qquad (1-\tau_{i^*})\vartheta_{i^*}(z) - (1-\tau_{i^*})\varphi\sum_k\vartheta_k(z)\Bigg] \\
& + \lambda z\Bigg[(1-\tau_i)\varphi\sum_k\vartheta_k'(z) - (1-\tau_i)\vartheta_i'(z) - (1-\tau_i)\sum_{k\neq i}\vartheta_k'(z) + (1-\tau_{i^*})\sum_{k\neq i^*}\vartheta_k'(z) + \\
& \qquad (1-\tau_{i^*})\vartheta_{i^*}'(z) - (1-\tau_{i^*})\varphi\sum_k\vartheta_k'(z)\Bigg].
\end{aligned} \tag{87}$$

Now plug the optimal labor $\ell_i = \left(\gamma p_i A_i (N_i z)^{\phi} w_i^{-1}\right)^{\frac{1}{1-\gamma}}$; then we can derive as before

$$\vartheta_k(z) = \Lambda_k \cdot z^{\frac{\phi+\gamma-1}{1-\gamma}},$$

and

$$\vartheta_k'(z) = \frac{\phi+\gamma-1}{1-\gamma}\Lambda_k \cdot z^{\frac{\phi+\gamma-1}{1-\gamma}-1}.$$

Plugging them back to the FOC of z and further simplifying:

$$\begin{aligned}
(1-\tau_i)\, p_i = & \sum_k (1-\tau_k)\Lambda_k z^{\frac{\phi+\gamma-1}{1-\gamma}} - \frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \left[\sum_{k\neq i,i^*} (1-\tau_k)\Lambda_k - (1-\tau_i)\sum_{k\neq i}\Lambda_k + (1-\tau_{i^*})\Lambda_{i^*}\right] \\
& - \frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \mathcal{C}(\lambda)(1-\tau_i)\sum_k \Lambda_k + \lambda\frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \Bigg[(1-\tau_i)\varphi\sum_k \Lambda_k - (1-\tau_i)\Lambda_k - (1-\tau_i)\sum_{k\neq i}\Lambda_k \\
& + (1-\tau_{i^*})\sum_{k\neq i^*}\Lambda_k + (1-\tau_{i^*})\Lambda_k - (1-\tau_{i^*})\varphi\sum_k \Lambda_k\Bigg] \\
= & \sum_k (1-\tau_k)\Lambda_k z^{\frac{\phi+\gamma-1}{1-\gamma}} - \frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \sum_k (\tau_i-\tau_k)\Lambda_k - \frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \mathcal{C}(\lambda)(1-\tau_i)\sum_k \Lambda_k \\
& - \lambda\frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} (\tau_{i^*}-\tau_i)(\varphi-1)\sum_k \Lambda_k.
\end{aligned}$$

The case of free transfer requires $\vartheta_k(z) = 0$ and the solution of z is the same as before

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)\, p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}. \tag{88}$$

The case of transfer pricing requires $\lambda = 0$, then

$$\begin{aligned}
(1-\tau_i)\, p_i = & \sum_k (1-\tau_k)\Lambda_k z^{\frac{\phi+\gamma-1}{1-\gamma}} \\
& - \frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \left[\sum_{k\neq i,i^*} (1-\tau_k)\Lambda_k - (1-\tau_i)\sum_{k\neq i}\Lambda_k + (1-\tau_{i^*})\Lambda_{i^*}\right] \\
= & \sum_k (1-\tau_k)\Lambda_k z^{\frac{\phi+\gamma-1}{1-\gamma}} - \frac{\phi}{1-\gamma} z^{\frac{\phi+\gamma-1}{1-\gamma}} \sum_k (\tau_i-\tau_k)\Lambda_k.
\end{aligned}$$

We can solve for z as:

$$z^{TP} = \left[\frac{\sum_k (1-\tau_k)\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (\tau_i-\tau_k)\Lambda_k}{(1-\tau_i)\, p_i}\right]^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{89}$$

thus, we obtain (84). Now turn to the PS case, we can solve for z as:

$$z^{PS} = \left[\frac{-\frac{\phi}{1-\gamma}\mathcal{C}(\lambda)\sum_k \Lambda_k}{p_i} + \frac{\sum_k (1-\tau_k)\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (\tau_i - \tau_k)\Lambda_k + \lambda\frac{\phi}{1-\gamma}(\tau_i - \tau_{i^*})(1-\varphi)\sum_k \Lambda_k}{(1-\tau_i)p_i} \right]^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{90}$$

thus, we obtain (85). ■

With these optimal intangible allocations derived, we now prove Lemma 1 under the alternative assumption. Note that we will not obtain the same elasticity results of z^{PS} with respect to φ and τ_{i^*} but rather show the comparative statics results hold, namely z^{PS} is decreasing in both φ and τ_{i^*}.

Proof of Proposition 1 under alternative assumption. Start from 1, it's obvious that when $\tau_i = \max\{\tau_1, ..., \tau_K\}$:

$$z^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)p_i} \right)^{\frac{1-\gamma}{1-\phi-\gamma}} > \left[\frac{\sum_k (1-\tau_k)\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (\tau_i - \tau_k)\Lambda_k}{(1-\tau_i)p_i} \right]^{\frac{1-\gamma}{1-\phi-\gamma}} = z^{TP}.$$

Now, towards showing 2. Start with ($\Leftarrow$) direction, and let $0 < \varphi < 1$. Then, by (80) we have $0 < \lambda < 1$. Take any $\lambda \in (0,1)$ and notice that $z^{PS} > z^{TP}$ iff

$$\mathcal{C}(\lambda) < \frac{\lambda(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i},$$

which has already been proven above. Given that all the inequalities are iffs the reverse argument holds immediately. Hence, we prove 2. To show 3. and 4. Notice from (80) first that $\frac{\partial \lambda}{\partial \varphi} < 0$. Now, we want to show

$$\begin{aligned}
\frac{\partial z^{PS}}{\partial \varphi} &= \frac{1-\gamma}{1-\phi-\gamma}\left(z^{PS}\right)^{\frac{\phi}{1-\gamma}} \frac{\sum_k \Lambda_k}{p_i}\frac{\phi}{1-\gamma}\left(\left(-\mathcal{C}'(\lambda)\frac{\partial \lambda}{\partial \varphi}\right) + \frac{\partial \lambda}{\partial \varphi}\left[\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right] - \lambda\frac{(\tau_i - \tau_{i^*})}{1-\tau_i}\right) \\
&= \frac{1-\gamma}{1-\phi-\gamma}\left(z^{PS}\right)^{\frac{\phi}{1-\gamma}} \frac{\sum_k \Lambda_k}{p_i}\frac{\phi}{1-\gamma}\left(\frac{\partial \lambda}{\partial \varphi}\left[\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i} - \mathcal{C}'(\lambda)\right] - \lambda\frac{(\tau_i - \tau_{i^*})}{1-\tau_i}\right) < 0,
\end{aligned}$$

which is true if

$$\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i} - \mathcal{C}'(\lambda) \le 0.$$

And it holds with equality, since it is the condition equalizing marginal cost with marginal benefit of profit shifting λ. Thus, we get

$$\frac{\partial z^{PS}}{\partial \varphi} = \frac{1-\gamma}{1-\phi-\gamma}\left(z^{PS}\right)^{\frac{\phi}{1-\gamma}} \frac{\sum_k \Lambda_k}{p_i}\frac{\phi}{1-\gamma}\left(-\lambda\frac{(\tau_i - \tau_{i^*})}{1-\tau_i}\right) < 0,$$

which proves 3. Notice, that proof for 4. follows analogously when Λ_{i^*} is sufficiently small, which is the empirically relevant case for us.

$$\begin{aligned}\frac{\partial z^{PS}}{\partial \tau_{i^*}} &= \frac{1-\gamma}{1-\phi-\gamma}\left(z^{PS}\right)^{\frac{\phi}{1-\gamma}}\left(\frac{-\Lambda_{i^*}+\frac{\phi}{1-\gamma}\Lambda_{i^*}-\lambda\frac{\phi}{1-\gamma}\left(1-\varphi\right)\sum_k\Lambda_k}{\left(1-\tau_i\right)p_i}\right)\\ &= \frac{1-\gamma}{1-\phi-\gamma}\left(z^{PS}\right)^{\frac{\phi}{1-\gamma}}\left(\frac{\frac{\phi+\gamma-1}{1-\gamma}\Lambda_{i^*}-\lambda\frac{\phi}{1-\gamma}\left(1-\varphi\right)\sum_k\Lambda_k}{\left(1-\tau_i\right)p_i}\right)<0.\end{aligned}$$

■

E.2 Allocation of Profit Rule

We first establish the following lemma characterizing how $\hat{\lambda}$ depends on the parameters of the profit allocation rule and how it differs from the share λ that is transferred under the existing tax regime. With slight abuse of notation we denote in the appendix:

$$\hat{\tau}_i(\theta)=(1-\theta)\tau_i+\theta\sum_i\tau_i\cdot\frac{p_iy_i}{\sum_k p_ky_k}.$$

Lemma 4 *Under Assumption 1, the following hold:*

1. *the fraction of intangible capital sold to the tax haven under the profit allocation rule is smaller than under the current regime, i.e.,* $\hat{\lambda}<\lambda$*;*
2. $\hat{\lambda}$ *is decreasing in* θ *with elasticity given by*

$$\varepsilon_{\theta}^{\hat{\lambda}}=-\mathcal{C}'\left(\hat{\lambda}\right)\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\left(\frac{\theta}{1-\theta}\right)\frac{\left(1-\widehat{\tau}\right)}{1-\left(\left(1-\theta\right)\tau_i+\theta\widehat{\tau}\right)}<0; \tag{91}$$

3. $\hat{\lambda}$ *is decreasing in* τ_i*, and if the MNE's sales in the tax haven are sufficiently small, then*

$$\left|\varepsilon_{\tau_{i^*}}^{\hat{\lambda}}\right|>\left|\varepsilon_{\tau_{i^*}}^{\lambda}\right|. \tag{92}$$

The first part of the lemma establishes that less intangible capital is transferred to the tax haven under the profit allocation rule than under the existing tax regime. This can be seen by comparing (11) and (24). The second part shows that $\hat{\lambda}$ is a decreasing function of the fraction of residual profits allocated based on sales, θ. Finally, the third part shows that $\hat{\lambda}$ is a decreasing function of the tax haven's tax rate, τ_{i^*}, just like λ. Importantly, however, if the tax haven accounts for a sufficiently small share of the MNE's global sales—which is the relevant case in our quantitative analysis—$\hat{\lambda}$ is more responsive to τ_{i^*} than λ. This implies that the profit allocation rule dampens the effect of the second OECD/G20 pillar, the global minimum corporate income tax.

Proof of Lemma 4. Start with derivitation of the optimal λ in the case of profit reallocation. Recall that the profit maximization problem of the MNE is

$$
\begin{aligned}
\max_{z,\lambda,\{\ell_i\}_{i=1}^{I}} \; & (1-\tau_i(1-\theta))\Big(p_i\Big(A_i(N_i z)^{\phi}\ell_i^{\gamma}\Big) - w_i\ell_i - p_i z \\
& +z\Big[\varphi\lambda\sum_k \vartheta_k(z) - \lambda\vartheta_i(z) + (1-\lambda)\sum_{k\neq i}\vartheta_k(z) - C(\lambda)\sum_k \vartheta_k(z)\Big]\Big) \\
& +(1-\tau_{i^*}(1-\theta))\Big(p_{i^*}\Big(A_{i^*}(N_{i^*}z)^{\phi}\ell_{i^*}^{\gamma}\Big) - w_k\ell_{i^*} \\
& +z\Big[\lambda\sum_{k\neq i^*}\vartheta_k(z) - (1-\lambda)\vartheta_{i^*}(z) - \varphi\lambda\sum_k\vartheta_k(z)\Big]\Big) \\
& +(1-\tau_k(1-\theta))\sum_{k\neq i,i^*}\Big(p_k\Big(A_k(N_k z)^{\phi}\ell_k^{\gamma}\Big) - w_k\ell_k - \vartheta_k(z)z\Big) \\
& -(1-(1-\theta))\sum_i \tau_i\cdot\frac{p_i y_i}{\sum_k p_k y_k}\cdot\Big[\sum_k\Big(p_k\Big(A_k(N_k z)^{\phi}\ell_k^{\gamma}\Big) - w_k\ell_k\Big) - p_i z - C(\lambda)\sum_k\vartheta_k(z)\Big].
\end{aligned}
$$

Take the derivative with respect to λ:

$$
\begin{aligned}
\lambda : 0 = & (\varphi-1)\sum_k \vartheta_k(z)\left[(1-\tau_i(1-\theta)) - (1-\tau_{i^*}(1-\theta))\right] \\
& -(1-\tau_i(1-\theta))\sum_k\vartheta_k(z)\mathcal{C}'(\lambda) + \theta\sum_k\vartheta_k(z)\mathcal{C}'(\lambda)\cdot\sum_i\tau_i\cdot\frac{p_i y_i}{\sum_k p_k y_k},
\end{aligned}
$$

and rearranging we get

$$
\mathcal{C}'(\lambda)\cdot\left[(1-\tau_i(1-\theta)) - \theta\sum_i\tau_i\cdot\frac{p_i y_i}{\sum_k p_k y_k}\right] = (1-\theta)(\varphi-1)(\tau_{i^*}-\tau_i),
$$

and we can derive:

$$
\begin{aligned}
\mathcal{C}'(\lambda) &= (1-\varphi)\frac{(1-\theta)(\tau_i-\tau_{i^*})}{(1-\tau_i(1-\theta)) - \theta\sum_i\tau_i\cdot\frac{p_i y_i}{\sum_k p_k y_k}} \\
\lambda &= (\mathcal{C}')^{-1}\left[\frac{(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right].
\end{aligned}
$$

Parametrizing $\mathcal{C}(\lambda) = \lambda + (1-\lambda)\log(1-\lambda)$, we can solve for λ as:

$$
\hat{\lambda} = 1 - \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right). \tag{93}
$$

Now, compare equation (93) with it's counterpart under the current tax regime, which is given by

$$
\lambda = 1 - \exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right).
$$

Towards proving 1., pick any $0 < \theta \leq 1$. Then the following sequence of inequalities holds:

$$
\begin{aligned}
1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right) &> 1 - \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right) \\
-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i} &< -\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)} \\
\frac{1}{1-\tau_i} &> \frac{1-\theta}{1-\hat{\tau}_i(\theta)} \\
1 &> \frac{1-(1-\theta)\tau_i - \theta}{1-(1-\theta)\tau_i - \theta\sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}} \\
-\theta \sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k} &> -\theta \\
\sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k} &< 1.
\end{aligned}
$$

The last inequality holds, since $\tau_k < 1$ $\forall k$ and all sales shares are by construction less than one. This proves that $\hat{\lambda} < \lambda$. Now, towards showing 2, inspect how θ affects $\hat{\lambda}$, i.e.

$$
\begin{aligned}
\frac{\partial \hat{\lambda}}{\partial \theta} = &- \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right) \cdot \\
&(-1)\left(\frac{-(1-\varphi)(\tau_i - \tau_{i^*})[1-\hat{\tau}_i(\theta)] + (1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})\left[\tau_i - \sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}\right]}{[1-\hat{\tau}_i(\theta)]^2}\right) \\
= &-(1-\varphi)(\tau_i - \tau_{i^*}) \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right) \frac{1-\sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}}{[1-\hat{\tau}_i(\theta)]^2} < 0,
\end{aligned}
$$

and the elasticity is given

$$
\begin{aligned}
\varepsilon_{\theta}^{\hat{\lambda}} = \frac{\partial \hat{\lambda}}{\partial \theta}\frac{\theta}{\hat{\lambda}} &= -(1-\varphi)(\tau_i - \tau_{i^*}) \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right) \frac{1-\sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}}{[1-\hat{\tau}_i(\theta)]^2} \frac{\theta}{\hat{\lambda}} \\
&= -\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right) \mathcal{C}'\left(\hat{\lambda}\right) \frac{\theta}{1-\theta} \frac{1-\sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}}{1-\hat{\tau}_i(\theta)} < 0,
\end{aligned}
$$

where in the last equality we used the first order condition of the profit function with respect to $\hat{\lambda}$. Hence, we have established 2. Now, inspect how τ_{i^*} affects $\hat{\lambda}$ to prove 3. First, compute the relevant partial derivative

$$
\begin{aligned}
\frac{\partial \hat{\lambda}}{\partial \tau_{i^*}} = &- \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right) \cdot \\
&(-1)\left(\frac{-(1-\varphi)(1-\theta)[1-\hat{\tau}_i(\theta)] + (1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})\left[-\frac{p_{i^*} y_{i^*}}{\sum_k p_k y_k}\right]}{[1-\hat{\tau}_i(\theta)]^2}\right) < 0,
\end{aligned}
$$

and hence the elasticity

$$\varepsilon^{\hat{\lambda}}_{\tau_{i^*}} = -\exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right)\cdot$$
$$(-1)\left(\frac{-(1-\varphi)(1-\theta)[1-\hat{\tau}_i(\theta)]+(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})\left[-\frac{p_{i^*}y_{i^*}}{\sum_k p_k y_k}\right]}{[1-\hat{\tau}_i(\theta)]^2}\right)\frac{\tau_{i^*}}{\hat{\lambda}}$$
$$= -\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\frac{(1-\varphi)}{(1-\tau_i)}\tau_{i^*}\left[(1-\tau_i)(1-\theta)\left(\frac{[1-\hat{\tau}_i(\theta)]+(\tau_i-\tau_{i^*})\left(\frac{p_{i^*}y_{i^*}}{\sum_k p_k y_k}\right)}{[1-\hat{\tau}_i(\theta)]^2}\right)\right].$$

Suppose that the size of tax-haven is negligible i.e. $p_{i^*}y_{i^*}\approx 0$, then we have

$$\varepsilon^{\hat{\lambda}}_{\tau_{i^*}} = -\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\frac{(1-\varphi)}{(1-\tau_i)}\tau_{i^*}(1-\tau_i)(1-\theta)\left(\frac{1-\hat{\tau}_i(\theta)}{[1-\hat{\tau}_i(\theta)]^2}\right)$$
$$= \varepsilon^{\lambda}_{\tau_{i^*}}\cdot\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\left(\frac{1-\lambda}{\lambda}\right)^{-1}\left(\frac{1-\tau_i(1-\theta)-\theta}{1-\hat{\tau}_i(\theta)}\right).$$

We want to show that $\left|\varepsilon^{\hat{\lambda}}_{\tau_{i^*}}\right| > \left|\varepsilon^{\lambda}_{\tau_{i^*}}\right|$; it suffices to show that:

$$\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\left(\frac{1-\lambda}{\lambda}\right)^{-1}\left(\frac{1-\tau_i(1-\theta)-\theta}{[1-\hat{\tau}_i(\theta)]}\right) > 1.$$

We know that

$$\lambda = 1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)$$
$$\hat{\lambda} = 1-\exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right).$$

Plugging into the inequality we want to show, we have:

$$\left(\frac{\hat{\lambda}}{1-\hat{\lambda}}\right)\cdot\left(\frac{[1-\hat{\tau}_i(\theta)]}{1-\theta}\right) < \frac{\lambda}{1-\lambda}\cdot(1-\tau_i)$$
$$\frac{1-\exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right)}{\exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}\cdot\left(\frac{[1-\hat{\tau}_i(\theta)]}{1-\theta}\right) < \frac{1-\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}{\exp\left(-\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)}\cdot(1-\tau_i)$$
$$\left[\exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{(1-\hat{\tau}_i(\theta))/(1-\theta)}\right)-1\right]\cdot\left(\frac{[1-\hat{\tau}_i(\theta)]}{1-\theta}\right) < \left[\exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)-1\right]\cdot(1-\tau_i).$$

We have shown that

$$\frac{[1-\hat{\tau}_i(\theta)]}{1-\theta} > (1-\tau_i).$$

Therefore, the inequality holds if $f(x) = \left(\exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x}\right)-1\right)\cdot x$ is an decreasing function when $x>0$, i.e. $f^{'}(x) < 0,\ x > 0$. Taking the derivative of $f(x)$, we have

$$\begin{aligned} f^{'}(x) &= \exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x}\right) - 1 + \exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x}\right)\cdot(-1)\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x^2}\cdot x \\ &= \exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x}\right) - \exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x}\right)\cdot\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x} - 1 \end{aligned}$$

Now, let $y = \frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x} > 0$, it's straight forward to show that $g(y) = exp(y) - exp(y)\cdot y - 1 < 0,\ y > 0$, as $g(0) = 0$ and $g'(y) = -exp(y)\cdot y < 0,\ y > 0$.

■

We now move on to prove Proposition 2. We first derive the formulas for allocation of the intangible capital under the profit allocation rule. The following lemma summarizes them.

Lemma 5 *The allocations of intangible capital and share of shifted intangible capital under the profit allocation rule are as follows:*

$$\begin{aligned} \hat{z}^{FT} &= \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{[1-\hat{\tau}_i(\theta)]\,p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \\ \hat{z}^{TP} &= \left(\frac{\sum_k \Lambda_k}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \\ \hat{z}^{PS} &= \hat{z}^{TP}\left(1-\mathcal{C}\left(\hat{\lambda}\right) + \frac{(1-\theta)\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}. \end{aligned}$$

Proof of Lemma 5. The proof follows the same procedure as Lemma 2 . ■

With Lemma 5 at hand, we are in a position to prove Lemma 2.

Proof of Proposition 2. We begin with proving 1. The proof relies on the following sequence of iff inequalities:

$$\begin{aligned} \hat{z}^{PS} &< z^{PS} \\ \hat{z}^{TP}\left(1-\mathcal{C}\left(\hat{\lambda}\right) + \frac{(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i-\tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right)^{\frac{1-\gamma}{1-\phi-\gamma}} &< z^{TP}\left(1-\mathcal{C}(\lambda) + \frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}} \\ 1-\mathcal{C}\left(\hat{\lambda}\right) + \hat{\lambda}\left[\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i}\right]\frac{(1-\tau_i)(1-\theta)}{1-\hat{\tau}_i(\theta)} &< 1-\mathcal{C}(\lambda) + \frac{\lambda(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i} \\ \lambda\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{1-\tau_i} - \hat{\lambda}\frac{(1-\varphi)(\tau_i-\tau_{i^*})(1-\theta)}{1-\hat{\tau}_i(\theta)} &> \mathcal{C}(\lambda) - \mathcal{C}\left(\hat{\lambda}\right), \end{aligned}$$

where we have

$$\mathcal{C}(\lambda) \equiv \lambda + (1-\lambda)\log(1-\lambda)$$
$$\hat{\lambda} = 1 - \exp\left(-\frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}\right)$$
$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}\right).$$

To simplify notation, let's denote:

$$\hat{A} = \frac{(1-\varphi)(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)},$$
$$A = \frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1-\tau_i}.$$

Plugging back to the inequality we want to show, we have

$$1 - \exp(-A) + \exp(-A)(-A) - 1 + \exp\left(-\hat{A}\right) - \exp\left(-\hat{A}\right)\left(-\hat{A}\right) < (1-\exp(-A))A - \left(1 - \exp\left(-\hat{A}\right)\right)\hat{A}$$
$$-\exp(-A) + \exp\left(-\hat{A}\right) < A - \hat{A}$$
$$\hat{A} + \exp\left(-\hat{A}\right) < A + \exp(-A).$$

We have shown that $0 < \hat{A} < A,\ \theta > 0$, thus proving the inequality above amounts to prove that function $f(x) = x + \exp(-x)$ is monotonically increasing when $x > 0$. Taking its derivative we get:

$$f'(x) = 1 - \exp(-x) > 0,\ x > 0.$$

To prove 2., we start with the partial derivative with respect to θ, i.e.

$$\frac{\partial \widehat{z}^{PS}}{\partial \theta} = \left(\frac{1-\gamma}{1-\phi-\gamma}\right)\hat{z}^{TP}\left(1 - \mathcal{C}\left(\hat{\lambda}\right) + \frac{(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i - \tau_{i^*})}{[1-\hat{\tau}_i(\theta)]}\right)^{\frac{1-\gamma}{1-\phi-\gamma}-1}\cdot$$
$$\left[-\mathcal{C}'\left(\hat{\lambda}\right)\frac{\partial\hat{\lambda}}{\partial\theta} + \frac{\left[\left(-\hat{\lambda} + (1-\theta)\frac{\partial\hat{\lambda}}{\partial\theta}\right)(1-\varphi)(\tau_i - \tau_{i^*})\right](1-\hat{\tau}_i(\theta)) - \left(\left(\tau_i - \frac{\sum_k \Lambda_k \tau_k}{\sum_k \Lambda_k}\right)\right)(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i - \tau_{i^*})}{[(1-\hat{\tau}_i(\theta))]^2}\right]$$
$$= \left(\frac{1-\gamma}{1-\phi-\gamma}\right)\hat{z}^{PS}\left(1 - \mathcal{C}\left(\hat{\lambda}\right) + \frac{(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i - \tau_{i^*})}{[1-\hat{\tau}_i(\theta)]}\right)^{-1}\cdot\left\{\frac{\partial\hat{\lambda}}{\partial\theta}\left[\frac{(1-\theta)(1-\varphi)(\tau_i - \tau_{i^*})}{(1-\hat{\tau}_i(\theta))} - \mathcal{C}'\left(\hat{\lambda}\right)\right] + \right.$$
$$\left.\frac{-\hat{\lambda}(1-\varphi)(\tau_i - \tau_{i^*})(1-\hat{\tau}_i(\theta)) - \left(\left(\tau_i - \frac{\sum_k \Lambda_k \tau_k}{\sum_k \Lambda_k}\right)\right)(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i - \tau_{i^*})}{[(1-\hat{\tau}_i(\theta))]^2}\right\},$$

and notice that the FOC w.r.t. $\hat{\lambda}$ is given by

$$\mathcal{C}'\left(\hat{\lambda}\right) = (1-\varphi)\frac{(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)}.$$

Thus to evaluate the sign, we need to sign the following

$$\frac{-\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left(1-\hat{\tau}_i(\theta)\right)-\left(\left(\tau_i-\frac{\sum_k\Lambda_k\tau_k}{\sum_k\Lambda_k}\right)\right)\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)}{\left[\left(1-\hat{\tau}_i(\theta)\right)\right]^2}$$

$$=\frac{\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left[-1+\left(1-\theta\right)\tau_i-\left(1-\theta\right)\tau_i+\frac{\sum_k\Lambda_k\tau_k}{\sum_k\Lambda_k}\right]}{\left[\left(1-\hat{\tau}_i(\theta)\right)\right]^2}$$

$$=\frac{\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left[\frac{\sum_k\Lambda_k\tau_k}{\sum_k\Lambda_k}-1\right]}{\left[\left(1-\hat{\tau}_i(\theta)\right)\right]^2}<0,$$

thus we have established that

$$\frac{\partial\widehat{z}^{PS}}{\partial\theta}<0.$$

And the elasticity is

$$\varepsilon_\theta^{\hat{z}^{PS}}=\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\hat{z}^{PS}\left(1-\mathcal{C}\left(\hat{\lambda}\right)+\frac{\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)}{\left[1-\hat{\tau}_i(\theta)\right]}\right)^{-1}\left[\frac{\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left[\frac{\sum_k\Lambda_k\tau_k}{\sum_k\Lambda_k}-1\right]}{\left[\left(1-\hat{\tau}_i(\theta)\right)\right]^2}\right]\frac{\theta}{\widehat{z}^{PS}}$$

$$=\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\frac{\theta}{1-\theta}\left(\frac{1}{\left[1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right]}\right)\frac{\frac{\sum_k\Lambda_k\tau_k}{\sum_k\Lambda_k}-1}{\left[1-\hat{\tau}_i(\theta)\right]}$$

$$=\varepsilon_\theta^{\hat{\lambda}}\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\left(\frac{\hat{\lambda}}{\mathcal{C}(\hat{\lambda})(1-\hat{\lambda})}\right)\left(\frac{1}{1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}}\right)<0.$$

Now, to show 3. consider the partial derivative of $\hat{z}^{PS}$ with respect to τ_{i^*},

$$\frac{\partial\hat{z}^{PS}}{\partial\tau_{i^*}}=\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\hat{z}^{TP}\left(1-\mathcal{C}\left(\hat{\lambda}\right)+\frac{\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)}{\left[1-\hat{\tau}_i(\theta)\right]}\right)^{\frac{1-\gamma}{1-\phi-\gamma}-1}\cdot$$
$$\left[\frac{\partial\hat{\lambda}}{\partial\tau_{i^*}}\left(\frac{\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left(1-\theta\right)\left[1-\hat{\tau}_i(\theta)\right]}{\left[1-\hat{\tau}_i(\theta)\right]^2}-\mathcal{C}'\left(\hat{\lambda}\right)\right)+\right.$$
$$\left.\left(\frac{\theta\frac{\Lambda_{i^*}}{\sum_k\Lambda_k}\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)-\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left[1-\hat{\tau}_i(\theta)\right]}{\left[1-\hat{\tau}_i(\theta)\right]^2}\right)\right]$$
$$=\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\hat{z}^{PS}\left(1-\mathcal{C}\left(\hat{\lambda}\right)+\frac{\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)}{\left[1-\hat{\tau}_i(\theta)\right]}\right)^{-1}\cdot$$
$$\left[\frac{\partial\hat{\lambda}}{\partial\tau_{i^*}}\left(\frac{\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left(1-\theta\right)}{1-\left(\left(1-\theta\right)\tau_i+\theta\sum_k\tau_k\frac{\Lambda_k}{\sum_k\Lambda_k}\right)}-\mathcal{C}'\left(\hat{\lambda}\right)\right)+\right.$$
$$\left.\left(\frac{\theta\frac{\Lambda_{i^*}}{\sum_k\Lambda_k}\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)-\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left[1-\hat{\tau}_i(\theta)\right]}{\left[1-\hat{\tau}_i(\theta)\right]^2}\right)\right].$$

Notice that the FOC wrt to $\hat{\lambda}$ implies

$$\mathcal{C}'\left(\hat{\lambda}\right) = (1-\varphi)\frac{(1-\theta)(\tau_i - \tau_{i^*})}{1-\hat{\tau}_i(\theta)},$$

thus the elasticity becomes

$$\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}} = \frac{\tau_{i^*}}{\hat{z}^{PS}}\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\hat{z}^{PS}\left(1-\mathcal{C}\left(\hat{\lambda}\right)+\frac{(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i-\tau_{i^*})}{[1-\hat{\tau}_i(\theta)]}\right)^{-1}\cdot$$
$$\frac{\theta\frac{\Lambda_{i^*}}{\sum_k \Lambda_k}(1-\theta)\hat{\lambda}(1-\varphi)(\tau_i-\tau_{i^*})-(1-\theta)\hat{\lambda}(1-\varphi)[1-\hat{\tau}_i(\theta)]}{[1-\hat{\tau}_i(\theta)]^2}$$
$$=\tau_{i^*}\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\left(\frac{1}{\left[1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right]}\right)\left(\frac{\theta\left[\frac{\Lambda_{i^*}}{\sum_k \Lambda_k}(\tau_i-\tau_{i^*})\right]-(1-\hat{\tau}_i(\theta))}{(\tau_i-\tau_{i^*})(1-\hat{\tau}_i(\theta))}\right)$$
$$=\left(\frac{-\tau_{i^*}}{\tau_i-\tau_{i^*}}\right)\left(\frac{1-\gamma}{1-\phi-\gamma}\right)\left(\frac{1}{\left[1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right]}\right)\left(\frac{1-\tau_i-\theta\left[\frac{\Lambda_{i^*}}{\sum_k \Lambda_k}(\tau_i-\tau_{i^*})+\sum_k \tau_k\frac{\Lambda_k}{\sum_k \Lambda_k}-\tau_i\right]}{(1-\hat{\tau}_i(\theta))}\right).$$

Compare it to the elasticity of z^{PS}

$$\varepsilon_{\tau_{i^*}}^{z^{PS}} = \left(\frac{1-\gamma}{1-\phi-\gamma}\right)\left(\frac{-\tau_{i^*}}{\tau_i-\tau_{i^*}}\right)\frac{1}{\left[1+\frac{1-\mathcal{C}(\lambda)}{\lambda\mathcal{C}'(\lambda)}\right]},$$

and note that

$$\lim_{\theta\to 0}\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}} = \varepsilon_{\tau_{i^*}}^{z^{PS}}.$$

To show this, we have

$$\left(\frac{1-\tau_i-\theta\left[\frac{\Lambda_{i^*}}{\sum_k \Lambda_k}(\tau_i-\tau_{i^*})+\sum_k \tau_k\frac{\Lambda_k}{\sum_k \Lambda_k}-\tau_i\right]}{(1-\hat{\tau}_i(\theta))}\right) = \left(\frac{(1-\hat{\tau}_i(\theta))+\theta\left[\frac{\Lambda_{i^*}}{\sum_k \Lambda_k}(\tau_i-\tau_{i^*})\right]}{(1-\hat{\tau}_i(\theta))}\right),$$

and under the assumption that sales to tax haven are negligible we have

$$\lim_{\Lambda_{i^*}\to 0}\left(\frac{(1-\hat{\tau}_i(\theta))+\theta\left[\frac{\Lambda_{i^*}}{\sum_k \Lambda_k}(\tau_i-\tau_{i^*})\right]}{(1-\hat{\tau}_i(\theta))}\right) = 1.$$

Finally, we want to prove $\left|\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}}\right| < \left|\varepsilon_{\tau_{i^*}}^{z^{PS}}\right|$. It suffices to show that

$$\frac{1}{\left(1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right)} < \frac{1}{\left(1+\frac{1-\mathcal{C}(\lambda)}{\lambda\mathcal{C}'(\lambda)}\right)}.$$

We have

$$\hat{\lambda} < \lambda$$
$$\frac{1}{\hat{\lambda}} > \frac{1}{\lambda},$$

but also

$$\hat{\lambda} < \lambda$$
$$\mathcal{C}\left(\hat{\lambda}\right) < \mathcal{C}\left(\lambda\right)$$
$$1 - \mathcal{C}\left(\hat{\lambda}\right) > 1 - \mathcal{C}\left(\lambda\right),$$

and also

$$\mathcal{C}'\left(\lambda\right) = -\log\left(1-\lambda\right) > 0$$
$$\mathcal{C}''\left(\lambda\right) = \frac{1}{1-\lambda} > 0.$$

Hence the marginal cost function is increasing in λ, therefore

$$\mathcal{C}'\left(\hat{\lambda}\right) < \mathcal{C}'\left(\lambda\right)$$
$$\frac{1}{\mathcal{C}'\left(\hat{\lambda}\right)} > \frac{1}{\mathcal{C}'\left(\lambda\right)}.$$

Therefore, we have that

$$\left(\frac{1}{\hat{\lambda}}\right)\left(1 - \mathcal{C}\left(\hat{\lambda}\right)\right)\left(\frac{1}{\mathcal{C}'\left(\hat{\lambda}\right)}\right) > \left(\frac{1}{\lambda}\right)\left(1 - \mathcal{C}\left(\lambda\right)\right)\left(\frac{1}{\mathcal{C}'\left(\lambda\right)}\right),$$

implying

$$\left(1 + \frac{1 - \mathcal{C}\left(\hat{\lambda}\right)}{\hat{\lambda}\mathcal{C}'\left(\hat{\lambda}\right)}\right) > \left(1 + \frac{1 - \mathcal{C}\left(\lambda\right)}{\lambda\mathcal{C}'\left(\lambda\right)}\right).$$

For any $\lambda \in (0, 1)$, we can show that

$$1 + \frac{1 - \mathcal{C}\left(\lambda\right)}{\lambda\mathcal{C}'\left(\lambda\right)} = \frac{\lambda\mathcal{C}'(\lambda) + 1 - \mathcal{C}(\lambda)}{\lambda\mathcal{C}'(\lambda)} = \frac{1 - \lambda - \log(1-\lambda)}{-\lambda \cdot \log(1-\lambda)} > 0.$$

Therefore,

$$\frac{1}{\left(1 + \frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right)} < \frac{1}{\left(1 + \frac{1-\mathcal{C}(\lambda)}{\lambda\mathcal{C}'(\lambda)}\right)},$$

and hence

$$\left|\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}}\right| < \left|\varepsilon_{\tau_{i^*}}^{z^{PS}}\right|.$$

■

E.2.1 Alternative Assumption

Here, we assume that MNEs internalize the effect of changing z on the licensing fee $\vartheta_k(z)$ and solve for optimal z under different scenarios (FT, TP, and PS). We then prove Lemma 2 under this assumption. As before, we start from the optimal z.

Lemma 6 *The allocations of intangible capital are as follows:*

$$\hat{z}^{FT} = \left(\frac{\sum_k (1-\tau_k)\Lambda_k}{(1-\tau_i)p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{94}$$

$$\hat{z}^{TP} = \left(\frac{\sum_k (1-\hat{\tau}_k(\theta))\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (1-\theta)(\tau_i-\tau_k)\Lambda_k}{(1-\hat{\tau}_i(\theta))p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}, \tag{95}$$

$$\hat{z}^{PS} = \left(\frac{-\frac{\phi}{1-\gamma}\mathcal{C}\left(\hat{\lambda}\right)\sum_k \Lambda_k}{p_i} + \frac{\sum_k (1-\hat{\tau}_k(\theta))\Lambda_k - \frac{\phi}{1-\gamma}\sum_k (1-\theta)(\tau_i-\tau_k)\Lambda_k + \hat{\lambda}\frac{\phi}{1-\gamma}(1-\theta)(\tau_i-\tau_{i^*})(1-\varphi)\sum_k \Lambda_k}{(1-\hat{\tau}_i(\theta))p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}. \tag{96}$$

Proof of Lemma 6. The proof follows the same procedure as the one of Lemma 3 ∎

We are now ready to prove Lemma 2.

Proof of Lemma 2 under alternative assumption. We start from proving 1 from deriving a set of iff inequalities:

$$\begin{aligned}
&\frac{\sum_k \Lambda_k}{p_i} - \frac{\phi+\gamma-1}{1-\gamma}\frac{\sum_k (1-\theta)(\tau_i-\tau_k)\Lambda_k}{(1-\hat{\tau}_i(\theta))p_i} - \left(1+\frac{\phi+\gamma-1}{1-\gamma}\right)\frac{\sum_k \Lambda_k}{p_i}\left[\mathcal{C}\left(\hat{\lambda}\right) - \frac{\hat{\lambda}(1-\theta)(\tau_i-\tau_{i^*})(1-\varphi)}{(1-\hat{\tau}_i(\theta))}\right] \\
&< \frac{\sum_k \Lambda_k}{p_i} - \frac{\phi+\gamma-1}{1-\gamma}\frac{\sum_k (\tau_i-\tau_k)\Lambda_k}{(1-\tau_i)p_i} - \left(1+\frac{\phi+\gamma-1}{1-\gamma}\right)\frac{\sum_k \Lambda_k}{p_i}\left[\mathcal{C}(\lambda) - \frac{\lambda(\tau_i-\tau_{i^*})(1-\varphi)}{(1-\tau_i)p_i}\right] \\
&-\frac{1-\phi-\gamma}{1-\gamma}\frac{\sum_k (\tau_i-\tau_k)\Lambda_k}{(1-\tau_i)p_i} + \frac{\phi}{1-\gamma}\frac{\sum_k \Lambda_k}{p_i}\left[\mathcal{C}(\lambda) - \frac{\lambda(\tau_i-\tau_{i^*})(1-\varphi)}{(1-\tau_i)}\right] \\
&< -\frac{1-\phi-\gamma}{1-\gamma}\frac{\sum_k (1-\theta)(\tau_i-\tau_k)\Lambda_k}{(1-\hat{\tau}_i(\theta))p_i} + \frac{\phi}{1-\gamma}\frac{\sum_k \Lambda_k}{p_i}\left[\mathcal{C}\left(\hat{\lambda}\right) - \frac{\hat{\lambda}(1-\theta)(\tau_i-\tau_{i^*})(1-\varphi)}{(1-\hat{\tau}_i(\theta))}\right].
\end{aligned}$$

We have proven before that $\mathcal{C}(\lambda) - \frac{\lambda(\tau_i-\tau_{i^*})(1-\varphi)}{(1-\tau_i)} < \mathcal{C}\left(\hat{\lambda}\right) - \frac{\hat{\lambda}(1-\theta)(\tau_i-\tau_{i^*})(1-\varphi)}{(1-\hat{\tau}_i(\theta))}$. It suffices to prove that

$$-\frac{1-\phi-\gamma}{1-\gamma}\frac{\sum_k (\tau_i-\tau_k)\Lambda_k}{(1-\tau_i)p_i} < -\frac{1-\phi-\gamma}{1-\gamma}\frac{\sum_k (1-\theta)(\tau_i-\tau_k)\Lambda_k}{(1-\hat{\tau}_i(\theta))p_i},$$

which simplifies to $1 > \sum_k \tau_k \frac{\Lambda_k}{\sum_i \Lambda_i}$. Thus, we have proven 1. We now prove 2:

$$\begin{aligned}\frac{d\hat{z}^{PS}}{d\theta} &= \frac{1-\gamma}{1-\phi-\gamma}\left(\hat{z}^{PS}\right)^{\frac{\phi}{1-\gamma}}\frac{1}{p_i}\Bigg[-\frac{\phi}{1-\gamma}\mathcal{C}'\left(\hat{\lambda}\right)\frac{d\hat{\lambda}}{d\theta}\sum_k \Lambda_k + \left(\frac{1-\theta}{\left(1-\hat{\tau}_i\left(\theta\right)\right)^2}\hat{\tau}_i^{'}\left(\theta\right)-\frac{1}{1-\hat{\tau}_i\left(\theta\right)}\right)\cdot \\ &\quad \left(\frac{\phi}{1-\gamma}\hat{\lambda}\left(\tau_i-\tau_{i^*}\right)\left(1-\varphi\right)-\frac{\phi+\gamma-1}{1-\gamma}\sum_k\left(\tau_i-\tau_k\right)\Lambda_k\right)+ \\ &\quad \frac{d\hat{\lambda}}{d\theta}\frac{\phi}{1-\gamma}\frac{\left(1-\theta\right)\left(\tau_i-\tau_{i^*}\right)\left(1-\varphi\right)}{1-\hat{\tau}_i\left(\theta\right)}\sum_k\Lambda_k\Bigg] \\ &= \frac{1-\gamma}{1-\phi-\gamma}\left(\hat{z}^{PS}\right)^{\frac{\phi}{1-\gamma}}\frac{1}{p_i}\left(\frac{\sum_k \tau_k \frac{\Lambda_k}{\sum_i \Lambda_i}-1}{\left(1-\hat{\tau}_i\left(\theta\right)\right)^2}\right)\left(\frac{\phi}{1-\gamma}\lambda\left(\tau_i-\tau_{i^*}\right)\left(1-\varphi\right)+\frac{1-\phi-\gamma}{1-\gamma}\sum_k\left(\tau_i-\tau_k\right)\Lambda_k\right).\end{aligned}$$

We have shown that

$$\sum_k \tau_k \frac{\Lambda_k}{\sum_i \Lambda_i} - 1 < 0.$$

The other terms are all positive, thus we have proven 2. Now to prove 3 we can show that

$$\begin{aligned}\frac{\partial\hat{z}^{PS}}{\partial\tau_{i^*}} &= \left(\frac{1-\gamma}{1-\phi-\gamma}\right)\left(\hat{z}^{PS}\right)^{\frac{\phi}{1-\gamma}}\frac{1}{p_i}\Bigg\{-\frac{\phi}{1-\gamma}\mathcal{C}'\left(\hat{\lambda}\right)\frac{\partial\hat{\lambda}}{\partial\tau_{i^*}}\sum_k\Lambda_k+\frac{\phi}{1-\gamma}\frac{\sum_k\Lambda_k}{\left(1-\hat{\tau}_i\left(\theta\right)\right)^2}\cdot \\ &\quad \left[\left(\frac{\partial\hat{\lambda}}{\partial\tau_{i^*}}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left(1-\theta\right)\sum_k\Lambda_k-\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\right)\left(1-\hat{\tau}_i\left(\theta\right)\right)+\left(\theta\frac{\Lambda_{i^*}}{\sum_k\Lambda_k}\right)\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\right] \\ &\quad -\frac{1-\phi-\gamma}{1-\gamma}\left(1-\theta\right)\Lambda_{i^*}\frac{1-\tau_i}{\left(1-\hat{\tau}_i\left(\theta\right)\right)^2}\Bigg\}.\end{aligned}$$

We have shown in previous proof that the sum of first two terms in the big bracket is negative. It's clear that the last term is also negative. Hence we have proven 3, that is $\frac{\partial\hat{z}^{PS}}{\partial\tau_{i^*}} < 0$. ■

F Quantitative model

F.1 Firm's problem with no transfer pricing or profit shifting

F.1.1 Scale choice: the parent division

We start from the parent division of a firm $\omega \in \Omega_i$'s scale choice here. A parent division that produces for the domestic market and exports to a set of J_X regions chooses its scale and how to allocate its output across its markets. Note that this problem nests the problem for firms only producing for the domestic markets when

$J_X = \emptyset$. The parent division's problem can then be written as

$$\pi_i^D(a, z; J_X) = \max_{q_{ii},(q_{ij})_{j\in J_X},\ell} \left\{ p_{ii}(q_{ii})q_{ii} + \sum_{j\in J_X} p_{ij}(q_{ij}^X)q_{ij}^X - W_i\ell \right\},$$

$$\text{s.t} \quad q_{ii} + \sum_{j\in J_X} \xi_{ij}q_{ij}^X = y_i = A_i a(N_i z)^\gamma \ell^\phi.$$

The first-order conditions are

$$[q_{ij}] \quad \frac{\varrho - 1}{\varrho} P_j Q_j^{\frac{1}{\varrho}} q_{ij}^{-\frac{1}{\varrho}} = \lambda\xi_{ij},$$

$$[\ell] \quad W_i = \lambda\phi A_i a(N_i z)^\gamma \ell^{\phi-1},$$

where $\xi_{ii} = 1$. Rearrange to get

$$\frac{\varrho - 1}{\varrho} P_j Q_j^{\frac{1}{\varrho}} q_{ij}^{-\frac{1}{\varrho}} = \frac{\tau_{ij}W_i}{\phi A_i a(N_i z)^\gamma \ell^{\phi-1}}.$$

Then

$$q_{ij} = \left[\frac{\phi(\theta-1)}{\theta}\right]^\theta \left[\frac{P_j Q_j^{\frac{1}{\theta}} A_i a(N_i z)^\gamma \ell^{\phi-1}}{\tau_{ij}W_i}\right]^\theta = \left[\frac{P_j Q_j^{\frac{1}{\theta}}}{\tau_{ij}}\right]^\theta \left[\frac{\phi(\theta-1)}{\theta}\right]^\theta \left[\frac{A_i a(N_i z)^\gamma \ell^{\phi-1}}{W_i}\right]^\theta.$$

Plugging this back into the resource constraint, we have

$$\left[P_i^\theta Q_i + \sum_{j\in J_X} P_j^\theta \tau_j^{1-\theta} Q_j\right] \left[\frac{\phi(\theta-1)}{\theta}\right]^\theta \left[\frac{A_i a(N_i z)^\gamma \ell^{\phi-1}}{W_i}\right]^\theta = A_i a(N_i z)^\gamma \ell^\phi,$$

which simplifies to

$$\left[P_i^\theta Q_i + \sum_{j\in J_X} P_j^\theta \tau_j^{1-\theta} Q_j\right] \left[\frac{\phi(\theta-1)}{\theta}\right]^\theta W_i^{-\theta}(A_i a)^{\theta-1}(N_i z)^{\gamma(\theta-1)} = \ell^{\phi+\theta-\theta\phi}.$$

We can solve the optimal labor choice

$$\ell = \left\{ \left[P_i^\varrho Q_i + \sum_{j\in J_X} P_j^\varrho \tau_j^{1-\varrho} Q_j\right] \left[\frac{\phi(\varrho-1)}{\varrho}\right]^\varrho W_i^{-\varrho}(A_i a)^{\varrho-1}(N_i z)^{\gamma(\varrho-1)} \right\}^{\frac{1}{\phi+\varrho-\varrho\phi}}. \tag{97}$$

We can use the equations above to compute q_{ij}, p_{ij}, and $\pi_i^D(a, z; J_X)$.

F.1.2 Scale choice: foreign subsidiaries

Foreign subsidiaries are similar to domestic-only firms. They just choose scale to maximize profits from selling to the host market given the demand curve and production technology. The only difference is the presence of the FDI barrier σ_{ij}. The foreign subsidiary's problem is

$$\pi_{ij}^F(a,z) = \max_{q,\ell}\ p_{ij}(q)q - W_i\ell$$
$$= \max_{\ell}\ P_jQ_j^{\frac{1}{\varrho}}\left(\sigma_{ij}A_ja\right)^{\frac{\varrho-1}{\varrho}}\left(N_jz\right)^{\gamma\frac{\varrho-1}{\varrho}}\ell^{\phi\frac{\varrho-1}{\varrho}} - W_j\ell.$$

The FOC is

$$\phi\frac{\varrho-1}{\varrho}P_jQ_j^{\frac{1}{\varrho}}\left(\sigma_{ij}A_ja\right)^{\frac{\varrho-1}{\varrho}}\left(N_jz\right)^{\gamma\frac{\varrho-1}{\varrho}}\ell^{\phi\frac{\varrho-1}{\varrho}-1} = W_j.$$

The optimal ℓ is then

$$\ell = \left\{\left[\frac{\phi(\varrho-1)}{\varrho}\right]^{\varrho}(P_j/W_j)^{\varrho}Q_j\left(\sigma_{ij}A_ja\right)^{\varrho-1}\left(N_jz\right)^{\gamma(\varrho-1)}\right\}^{\frac{1}{\phi+\varrho-\phi\varrho}}. \tag{98}$$

We can use this to compute $q_{ij} = y_j = \sigma_{ij}A_ja(N_jz)^{\gamma}\ell^{\phi}$, $p_{ij} = P_jQ_j^{\frac{1}{\varrho}}q_{ij}^{-\frac{1}{\varrho}}$, and $\pi_j^F(a,z)$.

F.1.3 Technology choice

Now that we have $\pi_i^D(a,z;J_X)$ of the parent division and $\pi_{ij}^F(a,z)$ of foreign affiliates, $j\in J_F$, we can determine how much R&D to do taking J_F and J_X as given. Note that we can ignore the fixed costs of exporting and FDI for now:

$$\hat{d}_i(a;J_X,J_F) = \max_z\left\{(1-\tau_i)\left[\pi_i^D(a,z;J_X) - W_iz/A_i\right] + \sum_{j\in J_F}(1-\tau_j)\pi_{ij}^F(a,z)\right\}.$$

Using the solutions for labor, the parent corporation's output can be written as

$$y_{ii} = Aa(N_iz)^{\gamma}\left\{\left[P_i^{\varrho}Q_i + \sum_{j\in J_X}P_j^{\varrho}\tau_j^{1-\varrho}Q_j\right]\left[\frac{\phi(\varrho-1)}{\varrho}\right]^{\varrho}W^{-\varrho}(A_ia)^{\varrho-1}(N_iz)^{\gamma(\varrho-1)}\right\}^{\frac{\phi}{\phi+\varrho-\varrho\phi}}$$
$$= \left\{\left[P_i^{\varrho}Q_i + \sum_{j\in J_X}P_j^{\varrho}\tau_j^{1-\varrho}Q_j\right]\left[\frac{\phi(\varrho-1)}{\varrho}\right]^{\varrho}W^{-\varrho}\right\}^{\frac{\phi}{\phi+\varrho-\varrho\phi}}(A_ia)^{\frac{\varrho}{\phi+\varrho-\phi\varrho}}(N_iz)^{\frac{\gamma\varrho}{\phi+\varrho-\phi\varrho}}.$$

We can use the FOC for q_{ij} to write

$$\frac{P_jQ_j^{\frac{1}{\varrho}}q_{ij}^{-\frac{1}{\varrho}}}{P_kQ_k^{\frac{1}{\varrho}}q_{ik}^{-\frac{1}{\varrho}}} = \frac{\tau_{ij}}{\tau_{ik}} \Rightarrow q_{ij} = \left(\frac{\tau_{ij}}{\tau_{ik}}\right)^{-\varrho}\left(\frac{P_j}{P_k}\right)^{\varrho}\left(\frac{Q_j}{Q_k}\right)q_{ik}.$$

Set $k=i$ and combine this with the resource constraint to get

$$q_{ii} + \sum_{j\in J_X}\tau_{ij}^{1-\varrho}\left(\frac{P_j}{P_i}\right)^{\varrho}\left(\frac{Q_j}{Q_i}\right)q_{ii} = y_{ii} \Rightarrow q_{ii} = \left(\frac{1}{1+\sum_{j\in J_X}\tau_{ij}^{1-\varrho}\left(\frac{P_j}{P_i}\right)^{\varrho}\left(\frac{Q_j}{Q_i}\right)}\right)y_{ii} = \bar{Q}_{ii}y_{ii}.$$

We can then write

$$q_{ij} = \tau_{ij}^{-\varrho} \left(\frac{P_j}{P_i} \right)^{\varrho} \left(\frac{Q_j}{Q_i} \right) \bar{Q}_{ii} y_{ii} = \bar{Q}_{ij} y_{ii}.$$

Then domestic parent revenues are

$$\begin{aligned} p_{ii} q_{ii} + \sum_{j \in J_X} p_{ij} q_{ij} &= P_i Q_i^{\frac{1}{\varrho}} q_{ii}^{\frac{\varrho-1}{\varrho}} + \sum_{j \in J_X} P_j Q_j^{\frac{1}{\varrho}} q_{ij}^{\frac{\varrho-1}{\varrho}} \\ &= \left[P_i Q_i^{\frac{1}{\varrho}} \bar{Q}_{ii}^{\frac{\varrho-1}{\varrho}} + \sum_{j \in J_X} P_j Q_j^{\frac{1}{\varrho}} \bar{Q}_{ij}^{\frac{\varrho-1}{\varrho}} \right] \\ &\qquad \times \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho - 1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\varrho\phi}} \\ &\qquad \times (A_i a)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} \\ &= \bar{R}_{ii} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}. \end{aligned} \tag{99}$$

Domestic parent costs are

$$\begin{aligned} W_i \ell + W_i z / A_i &= W_i \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho - 1)}{\varrho} \right]^{\varrho} W^{-\varrho} (A_i a)^{\varrho - 1} (N_i z)^{\gamma(\varrho-1)} \right\}^{\frac{1}{\phi+\varrho-\varrho\phi}} + W_i z / A_i \\ &= \bar{C}_{ii} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\varrho\phi}} + W_i z / A_i. \end{aligned}$$

Foreign affiliate revenues are

$$\begin{aligned} p_{ij} q_{ij} &= P_j Q_j^{\frac{1}{\varrho}} q_{ij}^{\frac{\varrho-1}{\varrho}} \\ &= \left[P_j Q_j^{\frac{1}{\varrho}} \right] \left[(P_j / W_j) \frac{\phi(\varrho - 1)}{\varrho} \right]^{\frac{\phi(\varrho-1)}{\phi+\varrho-\phi\varrho}} Q_j^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\phi\varrho}} (A_j \sigma_{ij} a)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_j^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} \\ &= \bar{R}_{ij} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}. \end{aligned} \tag{100}$$

Foreign affiliate costs are

$$\begin{aligned} W_j \ell &= W_j \left\{ \left[\frac{\phi(\varrho - 1)}{\varrho} \right]^{\varrho} (P_j / W_j)^{\varrho} Q_j \, (A_j \sigma_{ij} a)^{\varrho - 1} \, (N_j z)^{\gamma(\varrho-1)} \right\}^{\frac{1}{\phi+\varrho-\phi\varrho}} \\ &= \bar{C}_{ij} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}. \end{aligned}$$

Total net revenues are

$$(1 - \tau_i) p_{ii} q_{ii} + \sum_{j \in J_X} (1 - \tau_j) p_{ij} q_{ij} + \sum_{j \in J_F} p_{ij} q_{ij} = (1 - \tau_i) \bar{R}_{ii} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} + \sum_{j \in J_F} (1 - \tau_j) \bar{R}_{ij} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}.$$

Total costs are

$$(1-\tau_i)(W_i\ell_{ii}+W_iz/A_i)+\sum_{j\in J_F}(1-\tau_j)W_j\ell_{ij}=\left[(1-\tau_i)\bar{C}_{ii}+\sum_{j\in J_F}(1-\tau_j)C_{ij}\right]z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\varrho\phi}}+(1-\tau_i)W_iz/A_i.$$

We can write the objective function as

$$\left[(1-\tau_i)(\bar{R}_{ii}-\bar{C}_{ii})+\sum_{j\in J_F}(1-\tau_j)\left(\bar{R}_{ij}-\bar{C}_{ij}\right)\right]z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}-(1-\tau_i)W_iz/A_i.$$

The FOC is

$$\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\left[(1-\tau_i)(\bar{R}_{ii}-\bar{C}_{ii})+\sum_{j\in J_F}(1-\tau_j)\left(\bar{R}_{ij}-\bar{C}_{ij}\right)\right]z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}-1}=(1-\tau_i)W_i/A_i.$$

Then the optimal choice of z is

$$z=\left\{\left(\frac{\phi+\varrho-\phi\varrho}{\gamma(\varrho-1)}\right)\left[\frac{(1-\tau_i)W_i/A_i}{(1-\tau_i)\left(\bar{R}_{ii}-\bar{C}_{ii}\right)+\sum_{j\in J_F}(1-\tau_j)\left(\bar{R}_{ij}-\bar{C}_{ij}\right)}\right]\right\}^{\frac{\phi+\varrho-\phi\varrho}{\gamma\varrho+\phi\varrho-\gamma-\phi-\varrho}}.$$

F.1.4 Market choice

Now that we have $\hat{d}_i(a;J_X,J_F),\forall J_X,J_F$, we can decide where to export and where to operate foreign subsidiaries:

$$d_i(a)=\max_{J_X,J_F}\left\{\hat{d}_i(a;J_X,J_F)-W_i\left(\sum_{j\in J_X}\kappa_{jX}-\sum_{j\in J_F}\kappa_{jF})\right)\right\}. \tag{101}$$

This is a combinatorial discrete choice problem discussed in Arkolakis et al. (2021), as a firm's exporting and FDI choices interdependent. This problem is hard to solve since the number of potential decision sets grows exponentially in the number of regions. We limit the number of regions in the quantitative model to ease the computational burden.

F.2 Firm's problem with transfer pricing

Here, we solve the optimal nonrival technology allocation z in the environment with transfer pricing, taking J_X and J_F as given. We define total profit earned by a firm in this scenario as *transfer-pricing profit*, d_i^{TP}:

$$d_i^{TP}(a)=\max_{J_X,J_F}\left\{\hat{d}_i^{TP}(a;J_X,J_F)-W_i\left(\sum_{j\in J_X}\kappa_{jX}-\sum_{j\in J_F}\kappa_{jF})\right)\right\}, \tag{102}$$

where

$$\hat{d}_i^{TP}(a; J_X, J_F) = \max_z \Bigg\{ (1-\tau_i) \left[\pi_i^D(a, z; J_X) + (\sum_{j \in J_F} \vartheta_j(z) - W_i/A_i) z \right] + \sum_{j \in J_F} (1-\tau_j)(\pi_{ij}^F(a, z) - \vartheta_j(z) \cdot z) \Bigg\}. \tag{103}$$

Taking J_X and J_F as given, each firm chooses z to maximize $\hat{d}_i^{TP}(a; J_X, J_F)$. We can write the objective function as

$$\max_z \Bigg\{ (1-\tau_i) \left[(\bar{R}_{ii} - \bar{C}_{ii}) + \sum_{j \in J_F} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] + \sum_{j \in J_F} (1-\tau_j) \left(1 - \frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \Bigg\} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} - (1-\tau_i) W_i z / A_i.$$

The FOC is

$$\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) \Bigg\{ (1-\tau_i) \left[\bar{R}_{ii} - \bar{C}_{ii} + \sum_{j \in J_F} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] + \sum_{j \in J_F} (1-\tau_j) \left(1 - \frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \Bigg\} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} - 1} = (1-\tau_i) W_i / A_i.$$

Then the optimal choice of z is

$$z = \left\{ \left(\frac{\phi+\varrho-\phi\varrho}{\gamma(\varrho-1)} \right) \left[\frac{(1-\tau_i) W_i / A_i}{DENOM^{TP}} \right] \right\}^{\frac{\phi+\varrho-\phi\varrho}{\gamma\varrho+\phi\varrho-\gamma-\phi-\varrho}},$$

where

$$DENOM^{TP} = (1-\tau_i) \left[(\bar{R}_{ii} - \bar{C}_{ii}) + \sum_{j \in J_F} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] + \sum_{j \in J_F} (1-\tau_j) \left(1 - \frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}).$$

F.3 Firm's problem with transfer pricing and profit shifting

Here, we solve the optimal nonrival technology allocation z and profit shifting shares λ_{TH} and λ_{LT} in the environment with transfer pricing and profit shifting, taking J_X and J_F as given. This problem nests the one with only transfer pricing and no profit shifting simply by setting λ and $C(\lambda)$ to zero for both LT and TH. Here we solve for the full problem where λ_{LT} and $\lambda_{TH} > 0$. It is easier to rewrite $\hat{d}^{PS}(a)$ as:

$$d_i^{PS}(a) = \max_{J_X, J_F} \left\{ \hat{d}_i^{PS}(a; J_X, J_F) - W_i \left(\sum_{j \in J_X} \kappa_{jX} - \sum_{j \in J_F} \kappa_{jF} - \kappa_{iTH} \mathbb{1}(\lambda_{TH} > 0) \right) \right\},$$

where

$$\hat{d}_i^{PS} = \max_{z,\lambda_{LT},\lambda_{TH}} \Bigg\{(1-\tau_i)\Big[\pi_i^D(a,z;J_X)+\Big(-(\lambda_{LT}+\lambda_{TH})\vartheta_i(z)+(1-\lambda_{LT}-\lambda_{TH})\sum_{j\in J_F}\vartheta_j(z)$$

$$-W_i/A_i - W_i\big(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\big)\nu_i(z)\Big)z\Big]$$

$$+(1-\tau_{LT})\Big[\pi_{i,LT}^F(a,z)+\Big(\lambda_{LT}\sum_{j\in J_F\cup\{i\}\setminus\{LT\}}\vartheta_j(z)-(1-\lambda_{LT})\vartheta_{iLT}(z)\Big)z\Big]$$

$$+(1-\tau_{TH})\Big[\Big(\lambda_{TH}\sum_{j\in J_F\cup\{i\}}\vartheta_j(z)\Big)z\Big]$$

$$+\sum_{j\in J_F\setminus\{LT\}}(1-\tau_j)\left[\pi_{ij}^F(a,z)-\vartheta_j(z)z\right]\Bigg\}.$$

Substituting in the optimal scale choices specified in equation (97) and (98) and letting $\lambda=\lambda_{TH}+\lambda_{LT}$, we can write $\hat{d}_i^{PS}$ as

$$\max_{z,\lambda_{TH},\lambda_{LT}}\Bigg\{(1-\tau_i)\left[\left(1-\lambda\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\right)(\bar{R}_{ii}-\bar{C}_{ii})+(1-\lambda)\sum_{j\in J_F}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right]$$

$$-(1-\tau_i)\left[W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right]$$

$$+(1-\tau_{LT})\Bigg[\left(1-(1-\lambda_{LT})\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\right)(\bar{R}_{i,LT}-\bar{C}_{i,LT})+$$

$$\lambda_{LT}\sum_{j\in J_F\cup\{i\}\setminus\{LT\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\Bigg]$$

$$+(1-\tau_{TH})\left[\lambda_{TH}\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right]$$

$$+\sum_{j\in J_F\setminus\{LT\}}(1-\tau_j)\left[\left(1-\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right]\Bigg\}z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}-(1-\tau_i)W_i z/A_i.$$

And further simplifying

$$\max_{z,\lambda_{TH},\lambda_{LT}}\Bigg\{\sum_{j\in J_F\cup\{i\}}(1-\tau_j)(\bar{R}_{ij}-\bar{C}_{ij})-\sum_{j\in J_F\cup\{i\}}(\tau_i-\tau_j)\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})$$

$$+(\tau_i-\tau_{LT})\lambda_{LT}\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})$$

$$+(\tau_i-\tau_{TH})\lambda_{TH}\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})$$

$$-(1-\tau_i)W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\Bigg\}z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}-(1-\tau_i)W_i z/A_i.$$

Recall that the λ values can be solved independent of z:

$$\lambda_{LT} = \left(\mathcal{C}'_{i,LT}\right)^{-1}\left[\frac{1}{W_i}\frac{(\tau_i - \tau_{LT})}{1-\tau_i}\right],$$
$$\lambda_{TH} = \left(\mathcal{C}'_{i,TH}\right)^{-1}\left[\frac{1}{W_i}\frac{(\tau_i - \tau_{LT})}{1-\tau_i}\right].$$

The FOC for z is

$$\begin{aligned}(1-\tau_i)W_i/A_i = \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}-1}\Bigg\{ & \sum_{j\in J_F\cup\{i\}}(1-\tau_j)(\bar{R}_{ij}-\bar{C}_{ij}) \\ & -\sum_{j\in J_F\cup\{i\}}(\tau_i-\tau_j)\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\ & +(\tau_i-\tau_{LT})\lambda_{LT}\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\ & +(\tau_i-\tau_{TH})\lambda_{TH}\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\ & -(1-\tau_i)W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\Bigg\}.\end{aligned}$$

We can solve the optimal z as:

$$z = \left\{\left(\frac{\phi+\varrho-\phi\varrho}{\gamma(\varrho-1)}\right)\left[\frac{(1-\tau_i)\,W_i/A_i}{DENOM^{PS}}\right]\right\}^{\frac{\phi+\varrho-\phi\varrho}{\gamma\varrho+\phi\varrho-\gamma-\phi-\varrho}},$$

where $DENOM^{PS}$ is the stuff inside the big brackets above.

F.4 Profit allocation rule

As before, we solve for the full problem where $\lambda_{LT} > 0$ and $\lambda_{TH} > 0$. It's easier to state the firm's problem as:

$$d_i^{PR}(a) = \max_{z,J_X,J_F,\lambda_{TL},\lambda_{TH}}\left\{\hat{d}_i^{PR}(a;J_X,J_F) - W_i\left(\sum_{j\in J_X}\kappa_{jX} - \sum_{j\in J_F}\kappa_{jF} - \kappa_{iTH}\mathbb{1}(\lambda_{TH}>0)\right)\right\}.$$

Each firm, taking J_X and J_F as given, chooses z and λ to maximize

$$\hat{d}_i^{PR}(a;J_X,J_F,\varrho) = \max_{z,\lambda_{TH},\lambda_{LT}}\left\{\sum_{j\in\{i\}\cup J_X\cup J_F}\left(\pi_j^{PR}(a,z)-\tau_jT_j\right)\right\}, \tag{104}$$

where

$$\hat{d}_i^{PR} = \max_{z,\lambda_{LT},\lambda_{TH}} \Bigg\{ \Bigg[\pi_i^D(a,z;J_X) + \Bigg(-(\lambda_{LT}+\lambda_{TH})\vartheta_i(z) + (1-\lambda_{LT}-\lambda_{TH}) \sum_{j\in J_F} \vartheta_j(z) - W_i/A_i - W_i\big(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH})\big)\nu_i(z) \Bigg) z - \tau_i T_i \Bigg]$$
$$+ \Bigg[\pi_{i,LT}^F(a,z) + \Bigg(\lambda_{LT} \sum_{j\in J_F\cup\{i\}\setminus\{LT\}} \vartheta_j(z) - (1-\lambda_{LT})\vartheta_{iLT}(z) \Bigg) z - \tau_{LT} T_{LT} \Bigg]$$
$$+ \Bigg[\Bigg(\lambda_{TH} \sum_{j\in J_F\cup\{i\}} \vartheta_j(z) \Bigg) z - \tau_{TH} T_{TH} \Bigg]$$
$$+ \sum_{j\in J_F\setminus\{LT\}} \big[\pi_{ij}^F(a,z) - \vartheta_j(z) z - \tau_j T_j \big] + \sum_{j\in J_X\setminus J_F} [-\tau_j T_j] \Bigg\}.$$

T_j is the tax base in region j

$$\begin{aligned} T_j =& \Pi_j^r + (1-\theta)\cdot\Pi_j^R + \theta\cdot\frac{R_j}{\sum_k R_k}\cdot\Pi^R \\ &= \vartheta R_j + (1-\theta)\cdot(\pi_j(a,z;J_X) - \mu R_j) + \theta\cdot\frac{R_j}{\sum_k R_k}\cdot\sum_{k\in\{i\}\cup J_X\cup J_F}(\pi_k(a,z;J_X) - \mu R_k) \\ &= (1-\theta)\cdot\pi_j(a,z;J_X) + \theta\cdot\frac{R_j}{\sum_k R_k}\cdot\sum_{k\in\{i\}\cup J_X\cup J_F}\pi_k(a,z;J_X). \end{aligned}$$

Profit π_j is the profit earned in region j and it is zero if the firm does not operate in the region. Revenue earned in region j, denoted as R_j, include sales of both goods produced locally (by parent division or FDI) and goods exported to the region. Formally:

$$\begin{aligned} R_i &= p_{ii}(q_{ii})\,q_{ii}, \\ R_j &= p_{ij}^F(q_{ij})\,q_{ij}, j\in J_F, j\notin J_X, \\ R_j &= p_{ij}^X\left(q_{ij}^X\right)q_{ij}^X, j\in J_X, j\notin J_F, \\ R_j &= p_{ij}^F(q_{ij})\,q_{ij} + p_{ij}^X\left(q_{ij}^X\right)q_{ij}^X, j\in J_X\cap J_F, \\ R_j &= 0, \quad j\notin\{i\}\cup J_F\cup J_X. \end{aligned}$$

Thus, we can rewrite firm's problem as:

$$\hat{d}_i^{PR} = \max_{z,\lambda_{TH},\lambda_{LT}} \Bigg\{ \sum_{j\in\{i\}\cup J_X\cup J_F} \Bigg((1-\tau_j(1-\theta))\,\pi_j(a,z;J_X) - \tau_j\theta\cdot\frac{R_j}{\sum_j R_j}\cdot\sum_k \pi_k(a,z;J_X) \Bigg) \Bigg\}.$$

Further, substituting in π_i and denoting $\lambda = \lambda_{TH} + \lambda_{LT}$, we get

$$
\begin{aligned}
\max_{z,\lambda_{TH},\lambda_{LT}} \Bigg\{ & (1-(1-\theta)\tau_i) \left[\left(1-\lambda \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) \right) (\bar{R}_{ii}-\bar{C}_{ii}) + (1-\lambda) \sum_{j\in J_F} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \\
& -(1-(1-\theta)\tau_i) \left[W_i \left(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH}) \right) \sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \\
& +(1-(1-\theta)\tau_{LT}) \Bigg[\left(1-(1-\lambda_{LT}) \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) \right) (\bar{R}_{i,LT}-\bar{C}_{i,LT}) + \\
& \qquad \lambda_{LT} \sum_{j\in J_F\cup\{i\}\setminus\{LT\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij}-\bar{C}_{ij}) \Bigg] \\
& +(1-(1-\theta)\tau_{TH}) \left[\lambda_{TH} \sum_j \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \\
& + \sum_{j\in J_F\setminus\{LT\}} (1-\tau_j) \left[\left(1-\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \Bigg\} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} - (1-(1-\theta)\tau_i) W_i z / A_i \\
& - \sum_{j\in\{i\}\cup J_X\cup J_F} \tau_j \theta \cdot \frac{R_j}{\sum_j R_j} \cdot \sum_k \pi_k(a,z;J_X).
\end{aligned}
$$

Here we define $\tilde{R}_{ij}$ as the revenue shifter in region j for firms from region i, depending on region j is served. These terms are defined analogously of $\bar{R}_{ij}$ in equations (99) and (100).

$$
\begin{aligned}
\tilde{R}_{ii} &= P_i Q_i^{\frac{1}{\varrho}} \bar{Q}_{ii}^{\frac{\varrho-1}{\varrho}} \left\{ \left[P_i^{\varrho} Q_i + \sum_{j\in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho}\frac{\phi}{\phi+\varrho-\varrho\phi}} (A_i)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \\
\tilde{R}_{ij} &= P_j Q_j^{\frac{1}{\varrho}} \bar{Q}_{ij}^{\frac{\varrho-1}{\varrho}} \left\{ \left[P_i^{\varrho} Q_i + \sum_{j\in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho}\frac{\phi}{\phi+\varrho-\varrho\phi}} (A_i)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \quad j\in J_X, j\notin J_F, \\
\tilde{R}_{ij} &= \left[P_j Q_j^{\frac{1}{\varrho}} \right] \left[(P_j/W_j) \frac{\phi(\varrho-1)}{\varrho} \right]^{\frac{\phi(\varrho-1)}{\phi+\varrho-\phi\varrho}} Q_j^{\frac{\varrho-1}{\varrho}\frac{\phi}{\phi+\varrho-\phi\varrho}} (A_j\sigma_{ij})^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_j^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \quad j\in J_F, j\notin J_X, \\
\tilde{R}_{ij} &= \left[P_j Q_j^{\frac{1}{\varrho}} \right] \left[(P_j/W_j) \frac{\phi(\varrho-1)}{\varrho} \right]^{\frac{\phi(\varrho-1)}{\phi+\varrho-\phi\varrho}} Q_j^{\frac{\varrho-1}{\varrho}\frac{\phi}{\phi+\varrho-\phi\varrho}} (A_j\sigma_{ij})^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_j^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} \\
&\quad + P_j Q_j^{\frac{1}{\varrho}} \bar{Q}_{ij}^{\frac{\varrho-1}{\varrho}} \left\{ \left[P_i^{\varrho} Q_i + \sum_{j\in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho}\frac{\phi}{\phi+\varrho-\varrho\phi}} (A_i)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \quad j\in J_X\cap J_F, \\
\tilde{R}_{ij} &= 0, \quad j\notin\{i\}\cup J_F\cup J_X.
\end{aligned}
$$

With this definition, it's straightforward to show that the revenue share $\frac{R_j}{\sum_j R_j} = \frac{\tilde{R}_{ij}}{\sum_j \tilde{R}_{ij}}$. We can further obtain

$$
\begin{aligned}
\max_{z,\lambda_{TH},\lambda_{LT}} \Bigg\{ & (1-(1-\theta)\tau_i)\left[\left(1-\lambda\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\right)(\bar{R}_{ii}-\bar{C}_{ii})+(1-\lambda)\sum_{j\in J_F}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right] \\
& -(1-(1-\theta)\tau_i)\left[W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\sum_{j\in J_F\cup\{i\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right] \\
& +(1-(1-\theta)\tau_{LT})\Bigg[\left(1-(1-\lambda_{LT})\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\right)(\bar{R}_{i,LT}-\bar{C}_{i,LT})+ \\
& \qquad \lambda_{LT}\sum_{j\in J_F\cup\{i\}\setminus\{LT\}}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\Bigg] \\
& +(1-(1-\theta)\tau_{TH})\left[\lambda_{TH}\sum_{j}\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right] \\
& +\sum_{j\in J_F\setminus\{LT\}}(1-\tau_j)\left[\left(1-\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\right]\Bigg\} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}-(1-(1-\theta)\tau_i)W_i z/A_i \\
& -\sum_{j\in\{i\}\cup J_X\cup J_F}\tau_j\theta\cdot\frac{\tilde{R}_{ij}}{\sum_j\tilde{R}_{ij}}\cdot\Bigg\{\sum_k\left(\bar{R}_{ik}-\bar{C}_{ik}\right)z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}-W_i z/A_i- \\
& \qquad W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\cdot\sum_j\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}\Bigg\}.
\end{aligned}
$$

As before, the shift shares λ_{TH} and λ_{LT} can be solved independently of z:

$$
\begin{aligned}
\lambda_{TH} &= \left(\mathcal{C}'_{i,LT}\right)^{-1}\left[\frac{1}{W_i}\frac{(1-\theta)(\tau_i-\tau_{TH})}{1-(1-\theta)\tau_i-\theta\sum_k\tau_k\frac{\bar{R}_{ik}}{\sum_j\bar{R}_{ij}}}\right], \\
\lambda_{LT} &= \left(\mathcal{C}'_{i,TH}\right)^{-1}\left[\frac{1}{W_i}\frac{(1-\theta)(\tau_i-\tau_{LT})}{1-(1-\theta)\tau_i-\theta\sum_k\tau_k\frac{\bar{R}_{ik}}{\sum_j\bar{R}_{ij}}}\right].
\end{aligned}
$$

The FOC for z is

$$
\begin{aligned}
\left(1-(1-\theta)\tau_i-\theta\sum_j \tau_j \frac{\tilde{R}_{ij}}{\sum_k \tilde{R}_{ik}}\right) W_i/A_i = &\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}-1} \Bigg\{ \sum_{j\in J_F\cup\{i\}} (1-(1-\theta)\tau_j)(\bar{R}_{ij}-\bar{C}_{ij}) \\
&- \sum_{j\in J_F\cup\{i\}} (1-\theta)(\tau_i-\tau_j)\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\
&+ (1-\theta)(\tau_i-\tau_{LT})\lambda_{LT} \sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\
&+ (1-\theta)(\tau_i-\tau_{TH})\lambda_{TH} \sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\
&- (1-(1-\theta)\tau_i)W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\cdot \\
&\qquad \sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\
&- \sum_{j\in\{i\}\cup J_X\cup J_F} \tau_j\theta\cdot\frac{\tilde{R}_{ij}}{\sum_k \tilde{R}_{ik}}\cdot\Bigg[\sum_{k\in J_F\cup\{i\}} \left(\bar{R}_{ik}-\bar{C}_{ik}\right) \\
&- W_i(C_{i,TH}(\lambda_{TH})+C_{i,LT}(\lambda_{LT}))\sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\Bigg]\Bigg\}.
\end{aligned}
$$

Thus we can solve for optimal z as:

$$
z=\left\{\left(\frac{\phi+\varrho-\phi\varrho}{\gamma(\varrho-1)}\right)\left[\frac{\left(1-(1-\theta)\tau_i-\theta\sum_j \tau_j\frac{\tilde{R}_{ij}}{\sum_k \tilde{R}_{ik}}\right)W_i/A_i}{DENOM^{PR}}\right]\right\}^{\frac{\phi+\varrho-\phi\varrho}{\gamma\varrho+\phi\varrho-\gamma-\phi-\varrho}},
$$

where $DENOM^{PR}$ is the stuff inside the big brackets above.