

Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks

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Motivation

- **How should a government set fiscal instruments over time to deal with **inequality** and **individual risk**?**
- Want to provide a quantitative answer to this question.
- Need a model that is able to replicate realistic levels of inequality and individual (uninsurable) risk.
- The standard incomplete markets model has been relatively successful in this front.
- In this environment, we formulate a Ramsey problem and solve numerically for the **optimal transition** with 4 instruments:
 - linear capital and labor income taxes, lump-sum transfers (or taxes), and government debt.

This paper

- We do not maximize steady state welfare. Instead, the solution maximizes welfare along the transition between an initial and a final steady state:
 - The initial steady state we calibrate to replicate key features of the US economy;
 - The final steady state is endogenously determined and depends on the (time-varying) paths of the fiscal instruments.
- Taxation **distorts** agents decisions, but also affects the composition of agent's income in ways that allow the planner to provide **redistribution** and **insurance**.
- The resolution of the associated trade-offs determine the optimal policy.

Findings

- Capital income should be heavily taxed: staying at an imposed upper bound of 100% for 33 years, and converging to 45% in the limit.
- Labor income should also be taxed, but at lower levels: reduced to about 1/2 of its initial level to 13%.
- The (utilitarian) welfare gains, equivalent to a permanent 5% increase in consumption, come from
 - the redistribution implied by the higher capital taxes; and
 - the reduction in distortions from the lower labor taxes.
- Long-run capital taxes have to do with the provision of insurance.
- Accounting for transitory effects is important: implementing the policy that maximizes steady state welfare from the beginning leads to a welfare loss of 6% once the transitory effects are accounted for.

Mechanism: Two-Period Economy

Why use distortive capital and labor income taxes when non-distortive lump-sum taxes are available?

Two-Period Economy - Uncertainty Economy

- Continuum of ex-ante identical agents receive ω in period 1.
- In period 2 agents have random productivity levels:

$$e_L = 1 - \frac{\varepsilon}{\pi}, \quad e_H = 1 + \frac{\varepsilon}{1 - \pi}.$$

- No insurance market: only risk-free asset, a , available.
- Agents solve

$$\max_{a, c_L, c_H, n_L, n_H} u(\omega - a, \bar{n}) + \beta [\pi u(c_L, n_L) + (1 - \pi) u(c_H, n_H)]$$

$$\text{s.t. } c_i = (1 - \tau^n) w e_i n_i + (1 + (1 - \tau^k)r)a + T, \quad i = L, H.$$

- In period 2, firms choose K and N to maximize profits given a CRS production function $f(K, N)$, and prices w and r .

Two-Period Economy

Definition

The Ramsey problem is to choose τ^k , τ^n , and T to maximize welfare (the expected utility of the agents) subject to the economy being in equilibrium.

Assumption (A)

No income effects on labor supply and constant Frisch elasticity, i.e.

$$u_{cn} - u_{cc} \frac{u_n}{u_c} = 0, \quad \text{and} \quad \frac{u_{cc} u_n}{n(u_{cc} u_{nn} - u_{cn}^2)} = \kappa.$$

- In a similar setup, **Gottardi, Kajii and Nakajima (2014)** characterize the solution to the Ramsey problem with Assumption A replaced by assumptions about the sign of general equilibrium effects on prices.
- This assumption allows us to provide a sharper characterization.

Two-Period Economy - Uncertainty Economy

Proposition

In the uncertainty economy, if u satisfies Assumption A, then the optimal tax system is such that

$$\tau^n = \frac{(\nu - 1)\varepsilon}{(\nu - 1)\varepsilon + \kappa(\pi\nu - \pi + 1)} > 0, \text{ and}$$

$$\tau^k = 0,$$

where $\nu \equiv \frac{u_c(c_L, n_L)}{u_c(c_H, n_H)}$.

- **Insurance:** A positive labor income tax directly decreases the proportion of uncertain after tax labor income in total income.

Two-Period Economy - Inequality Economy

- Suppose now that productivity levels do not vary between agents, i.e. $e_L = e_H = 1$, but ω can take two values:

$$\omega_L = 1 - \frac{\epsilon}{\rho} \text{ (prop. } \rho), \quad \omega_H = 1 + \frac{\epsilon}{1 - \rho} \text{ (prop. } 1 - \rho).$$

Proposition

In the inequality economy, if u satisfies Assumption A and has CARA or CRRA, then the optimal (utilitarian) tax system is such that

$$\tau^k = \frac{(\nu - 1)\epsilon}{(\nu - 1)\epsilon + \frac{1}{\psi}\rho(\tau^k, r)(\pi\nu - \pi + 1)} > 0, \text{ and}$$

$$\tau^n = 0.$$

- **Redistribution:** A positive capital income tax directly decreases the proportion of unequal after tax capital income in total income.

Infinite-Horizon Model

Quantitative model in which we investigate the properties of the optimal policy

Environment - Households

- There is a measure one of households.
- Individual states: $a \in A$ - assets, and $e \in E$ - stochastic productivity that follows a Markov process with matrix Γ .
- Given a sequence of prices and taxes the household solves

$$v_t(a, e) = \max_{c_t, n_t, a_{t+1}} u(c_t, n_t) + \beta \sum_{e_{t+1} \in E} v_{t+1}(a_{t+1}, e_{t+1}) \Gamma_{e, e_{t+1}}$$

subject to

$$\begin{aligned} (1 + \tau^c) c_t(a, e) + a_{t+1}(a, e) &= (1 - \tau_t^n) w_t e n_t(a, e) + \\ &+ (1 + (1 - I_{\{a \geq 0\}} \tau_t^k) r_t) a + T_t \\ a_{t+1}(a, e) &\geq \underline{a}. \end{aligned}$$

Environment - Firm and Government

- Given prices, in each period, the representative firm solves

$$\max_{K_t, N_t} f(K_t, N_t) - w_t N_t - r_t K_t$$

- Government finances an exogenous stream of expenditure, and lump-sum transfers, with taxes on consumption, labor and capital, or debt

$$G + T_t + r_t B_t = B_{t+1} - B_t + \tau^c C_t + \tau_t^n w_t N_t + \tau_t^k r_t \hat{A}_t.$$

where \hat{A}_t is the tax base for the capital income tax.

Equilibrium

Definition

Given K_0, B_0 , an initial distribution λ_0 and a policy $\pi \equiv \{\tau_t^k, \tau_t^n, T_t\}_{t=0}^\infty$, a **competitive equilibrium** is a sequence of value functions $\{v_t\}_{t=0}^\infty$, an allocation $X \equiv \{c_t, n_t, a_{t+1}, K_{t+1}, N_t, B_{t+1}\}_{t=0}^\infty$, a price system $P \equiv \{r_t, w_t\}_{t=0}^\infty$, and a sequence of distributions $\{\lambda_t\}_{t=0}^\infty$, such that for all t :

- 1 Given P and π , $c_t(a, e)$, $n_t(a, e)$, and $a_{t+1}(a, e)$ solve the household's problem and $v_t(a, e)$ is the respective value function;
- 2 Factor prices are set competitively: $r_t = f_K(K_t, N_t)$,
 $w_t = f_N(K_t, N_t)$;
- 3 The probability measure λ_t is consistent with Γ and $a_{t+1}(a, e)$;
- 4 Government budget constraint holds and debt is bounded;
- 5 Markets clear,

$$C_t + G + K_{t+1} - K_t = f(K_t, N_t), \quad K_t + B_t = \int_{A \times E} a_t(a, e) d\lambda_t.$$

Procedure

- 1 We calibrate the initial stationary equilibrium to match data on macro aggregates, wealth inequality, statistics about the labor income process, and the current levels of the fiscal instruments.
- 2 Then, given this and paths for the fiscal instruments we can compute a transition to an endogenously determined final steady state.
- 3 Finally, we parametrize these paths and optimize in the space of sequences of fiscal instruments.

Calibration

Target Statistics and Parameters

- Preferences and technology:

$$u(c, n) = \frac{1}{1-\sigma} \left(c - \chi \frac{n^{1+\frac{1}{\kappa}}}{1+\frac{1}{\kappa}} \right)^{1-\sigma}, \quad f(K, N) = K^\alpha N^{1-\alpha} - \delta K$$

Statistic	Target	Model	Parameter	Value
Preferences and Technology				
Intertemporal elast. of subst.	-	0.50	σ	2.00
Frisch elasticity	-	0.72	κ	0.72
Average hours worked	0.30	0.30	χ	4.12
Capital to output	2.72	2.71	β	0.97
Capital income share	-	0.38	α	0.38
Investment to output	0.27	0.27	δ	0.10

*Exogenously imposed parameter.

Target Statistics and Parameters

Statistic	Target	Model	Parameter	Value
Borrowing Constraint				
Hh with negative wealth (%)	18.6	19.1	\underline{a}	-0.04
Fiscal Policy				
Capital income tax (%)	-	36.0	τ^k	0.36
Labor income tax (%)	-	28.0	τ^n	0.28
Consumption tax (%)	-	5.0	τ^c	0.05
Transfer to output (%)	8.0	8.0	T	0.08
Debt to Output (%)	63.0	63.0	G	0.15

- In order to set the tax rates in the initial steady state, we use the effective average tax rates (tax revenue over tax base for each source) computed by [Trabandt and Uhlig \(2011\)](#) for 1995-2007.

Target Statistics and Parameters

Statistic	Target	Model	Parameter	Value
Labor Productivity Process				
Wealth Gini index	0.82	0.81	e_1/e_2	0.62
% of wealth in 1st quintile	-0.2	-0.2	e_3/e_2	3.77
% of wealth in 4th quintile	11.2	10.2	Γ_{11}	0.96
% of wealth in 5th quintile	83.4	83.4	Γ_{12}	0.04
% of wealth in top 5%	60.3	60.8	Γ_{21}	0.07
Corr. btw wealth and lab. inc.	0.29	0.29	Γ_{22}	0.93
Autocorrelation of lab. inc.	0.90	0.90	Γ_{31}	0.01
Std. dev. of inn. to lab. inc.	0.20	0.20	Γ_{32}	0.05

$$e = \begin{bmatrix} 0.8 \\ 1.3 \\ 4.9 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} .96 & .04 & .00 \\ .07 & .93 & .00 \\ .01 & .05 & .94 \end{bmatrix}, \quad \text{and} \quad \pi^* = \begin{bmatrix} .616 \\ .377 \\ .007 \end{bmatrix}$$

Model Performance

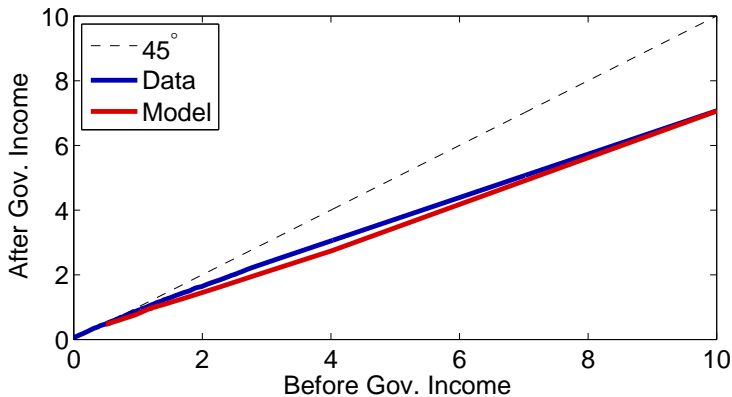
- Without targeting we approximate well the composition of income:

Quintile	Model			Data		
	Labor	Asset	Transfer	Labor	Asset	Transfer
1st	83.7	-0.1	16.4	82.0	2.0	16.0
2nd	85.4	1.6	13.1	83.0	4.8	12.2
3rd	84.1	4.7	11.2	80.0	7.3	12.7
4th	81.4	8.6	10.0	77.6	10.3	12.1
5th	58.7	36.2	5.2	51.7	40.0	8.3

- We also exactly match the consumption Gini, which for the period we calibrate the economy to remained virtually constant at **0.27** (see [Krueger and Perri \(2006\)](#)).

Model Performance: Income tax schedule

- The tax rates are calibrated to match effective tax rates. However, we also approximate well the actual income tax schedule (data from Heathcote, Storesletten & Violante (2014)).



Notes: The axis units are income relative to the mean.

Ramsey Problem

Ramsey Problem

Definition

Given λ_0 , K_0 , B_0 , τ_0^k , τ_0^n , T_0 and a welfare function W , the **Ramsey problem** is $\max_{\pi} W(X(\pi))$ subject to $X(\pi)$ being an equilibrium allocation and π satisfying $\tau_t^k \leq 1 \quad \forall t \geq 1$.

- The benchmark welfare function is utilitarian:

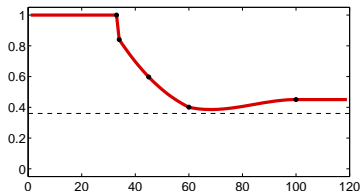
$$W(\pi) = \int_S E_0 \sum_{t=0}^{\infty} \beta^t u(c_t(a, e|\pi), n_t(a, e|\pi)) d\lambda_0.$$

- Solving this problem involves searching on the space of sequences $\{\tau_t^k, \tau_t^n, T_t\}_{t=1}^{\infty}$.
- In order to make it computationally feasible we parameterize these sequences imposing the ad-hock constraints that they are smooth and converge in the long-run.

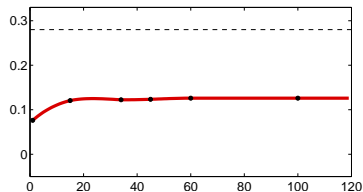
Results

Main result

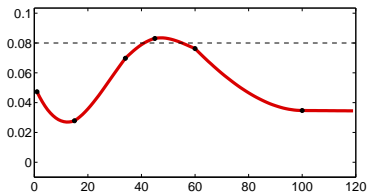
Capital tax



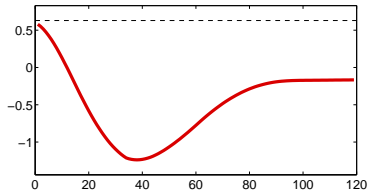
Labor tax



Lump sum to GDP



Debt to GDP

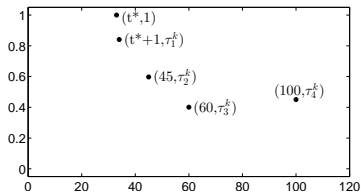


Enough Nodes

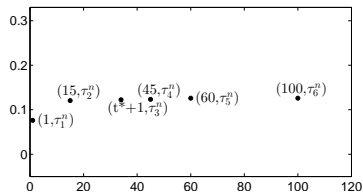
Aggregates

Main result

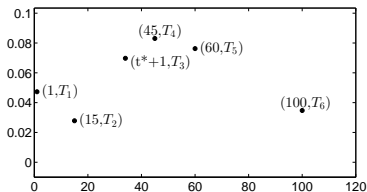
Capital tax



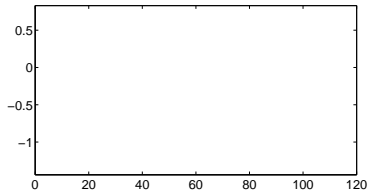
Labor tax



Lump sum to GDP



Debt to GDP

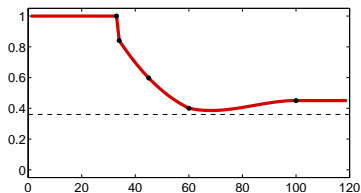


Enough Nodes

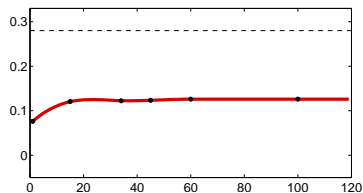
Aggregates

Main result

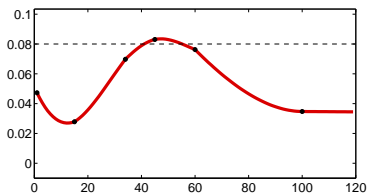
Capital tax



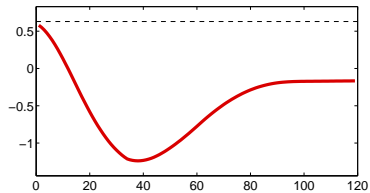
Labor tax



Lump sum to GDP

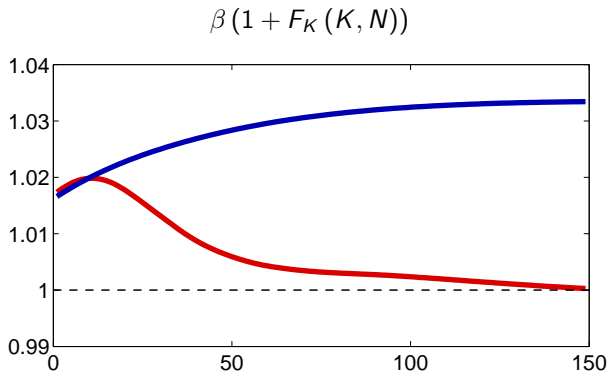


Debt to GDP



- Average welfare gain equivalent to a permanent 4.9% increase in consumption.

Modified golden rule holds in the final steady state



Red line: Benchmark; Blue line: Economy with constant optimal policy over transition.

Other Long-run Optimality Conditions

Main result

- Relative to keeping fiscal instruments at their initial levels, this plan leads to an average welfare gain equivalent to a 4.9% increase in consumption in every state and date.
- Conditional on the initial type (a, e), what percentage of the population would find the reform beneficial?

All	$e = L$	$e = M$	$e = H$
99.5	99.6	98.3	3.7

- Low and middle productivity agents gain. Cross section picture
- Income composition changes: share of labor income increases and asset income decreases: more risk, but less inequality.

Income composition dynamics

Welfare Decomposition

We can decompose the welfare gains into what comes from the reduction in distortions, redistribution, and insurance

Welfare decomposition

- The utilitarian welfare function can increase for three reasons:
 - 1 Reduction in distortions, if the utility of the average agent, $U(C_t, N_t)$, increases: **the level effect (Δ_L)**;
 - 2 Transfers from ex-post rich to ex-post poor, if the uncertainty of each individual path $\{c_t, n_t\}_{t=1}^{\infty}$ is reduced: **the insurance effect (Δ_I)**;
 - 3 Transfers from ex-ante rich to ex-ante poor, if the inequality between certainty equivalents for $\{c_t, n_t\}_{t=1}^{\infty}$ is reduced: **the redistribution effect (Δ_R)**.

Proposition

Let Δ be the utilitarian (average) welfare gain. The following decomposition holds (see *Flodén (2001)* and *Bénabou (2002)*):

$$(1 + \Delta) = (1 + \Delta_L)(1 + \Delta_R)(1 + \Delta_I)$$

Δ	Δ_L	Δ_R	Δ_I
4.9	3.7	4.9	-3.7

Fixed Instruments

Fixing each instrument at their initial level we can understand their contribution

Fixed instruments - Welfare decomposition

	Average welfare gain Δ	Level effect Δ_L	Redistribution effect Δ_R	Insurance effect Δ_I
Fixed capital taxes	1.0	3.7	-0.2	-2.5
Fixed labor taxes	3.3	0.0	4.8	-1.6
Fixed lump-sum	4.4	1.8	5.1	-2.5
Fixed debt	4.0	3.8	3.2	-3.2
Benchmark	4.9	3.7	4.9	-3.7

- The gains from redistribution are achieved via higher capital taxes in the initial periods. Long-run Capital Taxes
- The reductions in distortions are achieved by the reduction in labor taxes.
- Movements in transfers and debt are relatively less important. Gov. Debt

Transitory effects

Ignoring transitory effects can be severely misleading

Transitory effects are important

	Labor tax τ^h	Capital tax τ^k	Transfers T/Y	Debt B/Y
Initial equilibrium	28.0	36.0	8.0	63.0
Ignoring transition	4.7	-5.2	-5.4	63.0
Benchmark (long-run)	12.6	45.1	3.5	-16.9

- Implementing the policy that maximizes steady state welfare accounting for its transitory effects leads to an average welfare **loss of 6.4%**.
- Importantly, the transitory distributional effects of the policy and the costs associated with the accumulation of capital are ignored.

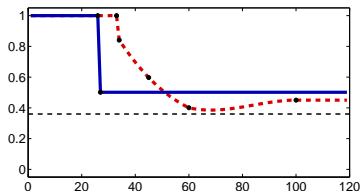
Constant Policy

Inequality Aversion

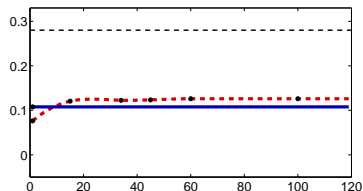
Alternative welfare functions

A good approximation

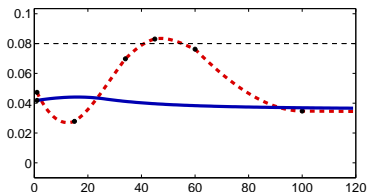
Capital tax



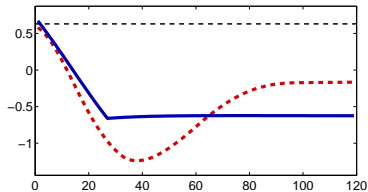
Labor tax



Lump sum to GDP



Debt to GDP



- The solution with 3 choices produces a reasonable approximation for the benchmark solution. Welfare gains: 4.64 vs. 4.90.

Welfare functions

- The utilitarian welfare function implies a particular social preference with respect to the equality versus efficiency trade-off.
- To rationalize different preferences about this trade-off, we use the following welfare function¹

$$W^{\hat{\sigma}} = \left(\int \bar{x}(a_0, e_0)^{1-\hat{\sigma}} d\lambda_0 \right)^{\frac{1}{1-\hat{\sigma}}}$$

- If $\hat{\sigma} = \sigma$, $W^{\hat{\sigma}}$ is equivalent to the utilitarian welfare function.
- If $\hat{\sigma} = 0$, maximizing $W^{\hat{\sigma}}$ is equivalent to maximizing $(1 + \Delta_L)(1 + \Delta_I)$, i.e. the planner has no equality concerns.
- As $\hat{\sigma} \rightarrow \infty$, $W^{\hat{\sigma}}$ approaches $\min(\bar{x}(a_0, e_0))$.

¹where \bar{x} denotes the certainty equivalent of a consumption-labor composite.

Different degrees of inequality aversion

- Higher $\hat{\sigma}$ imply higher t^* 's. However, the final levels of capital and labor taxes are remarkably unaffected by changes in this parameter.

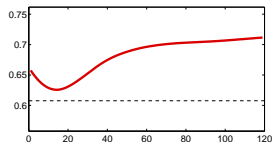
	t^*	τ^k	τ^n	T/Y	B/Y
$\hat{\sigma} = 0$	0	34.7	12.2	0.0	79.8
$\hat{\sigma} = 1$	19	49.9	10.1	2.9	-36.4
$\hat{\sigma} = 2^*$	26	49.7	10.8	3.6	-62.5
$\hat{\sigma} = 3$	29	49.8	10.4	3.5	-76.8
$\hat{\sigma} = 4$	30	48.9	11.5	4.1	-76.0
$\hat{\sigma} = 5$	32	49.2	11.3	4.0	-84.2

Conclusions

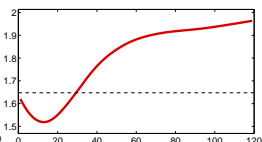
- Numerically, we are able to approximate the solution of the Ramsey problem in the standard incomplete markets model.
- We find that capital income taxes should be used more and labor taxes less than they are in the US:
 - Short-run: redistribution; transfer from ex-ante rich to poor.
 - Long-run: an agent's current asset level is a better estimate of how lucky it has been than its current labor income.
- We also show that accounting for the transitory effects of policy is important.
- Introducing human capital or life cycle, for instance, would only be harder to the extent that it takes longer to compute the transition.

Aggregates

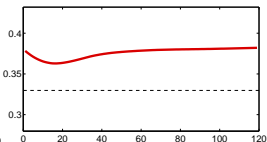
Output



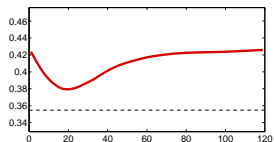
Capital



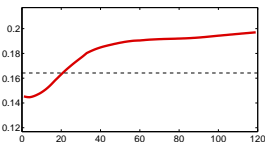
Labor



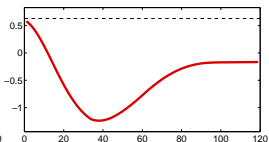
Consumption



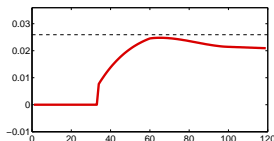
Investment



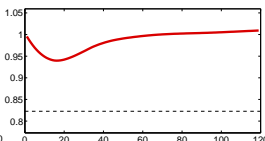
Debt to GDP



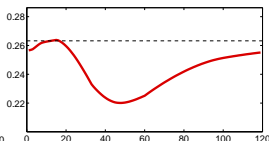
After tax int. rate



After tax wage

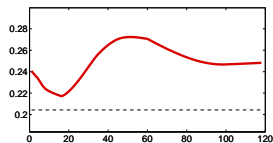


Consumption Gini

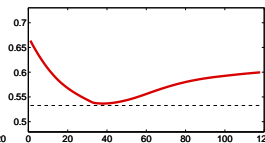


Cross sectional effects

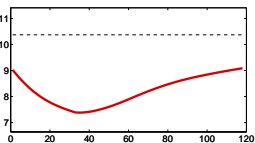
Consumption (L)



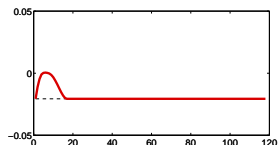
Consumption (M)



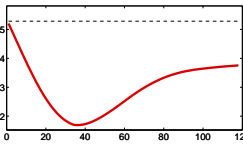
Consumption (H)



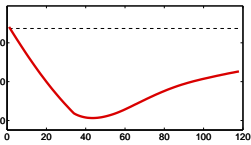
Assets (L)



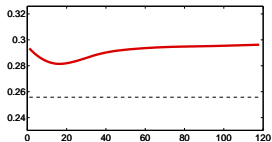
Assets (M)



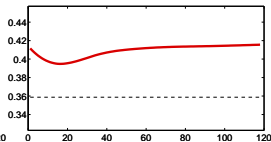
Assets (H)



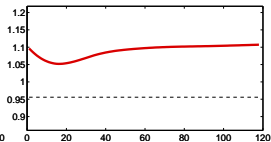
Labor (L)



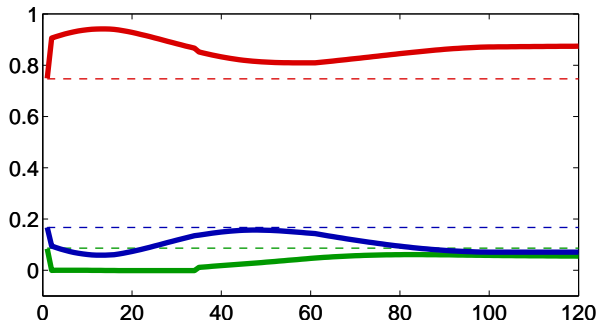
Labor (M)



Labor (H)



Income composition dynamics



Notes: Shares in the after-tax total income (dashed: no reform, solid: optimal transition). Red lines: labor income share; Blue lines: transfer income share; Green lines: asset income share

Welfare Decomposition

- Average welfare gain, Δ :

$$\int E_0 [U(\{x_t^R\})] d\lambda_0(a_0, e_0) = \int E_0 [U((1 + \Delta)\{x_t^{NR}\})] d\lambda_0(a_0, e_0),$$

where λ_0 is the initial distribution of initial states (a_0, e_0) .

- Aggregate consumption-labor composite, X_t^j :

$$X_t^j \equiv \int x_t^j(a_0, e_0) d\lambda_0(a_0, e_0), \quad \text{for } j = R, NR$$

- Individual certainty equivalent, $\bar{x}^j(a_0, e_0)$:

$$U(\{\bar{x}^j\}) \equiv E_0 [U(\{x_t^j\})], \quad \text{for } j = R, NR.$$

- Aggregate certainty equivalent, \bar{X}^j :

$$\bar{X}^j \equiv \int \bar{x}^j(a_0, e_0) d\lambda_0(a_0, e_0), \quad \text{for } j = R, NR.$$

Welfare Decomposition

- Level Effect, Δ_L :

$$U(\{X_t^R\}) = U((1 + \Delta_L)\{X_t^{NR}\}).$$

- Uncertainty Effect, Δ_U :

$$U\left(\left(1 - p_{unc}^j\right)\{X_t^j\}\right) = U\left(\{\bar{X}_t^j\}\right), \text{ then,}$$

$$1 + \Delta_U = \frac{1 - p_{unc}^R}{1 - p_{unc}^{NR}}.$$

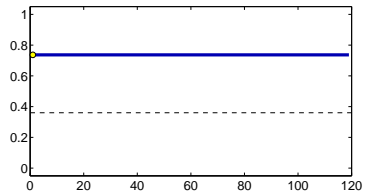
- Inequality Effect, Δ_I :

$$U\left(\left(1 - p_{ine}^j\right)\{\bar{X}_t^j\}\right) = \int U(\{\bar{x}^j(a_0, e_0)\}) d\lambda(a_0, e_0), \text{ then,}$$

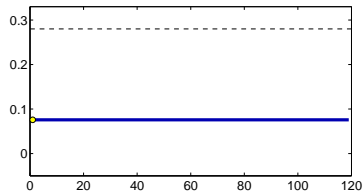
$$1 + \Delta_I = \frac{1 - p_{ine}^R}{1 - p_{ine}^{NR}}.$$

2 Choices

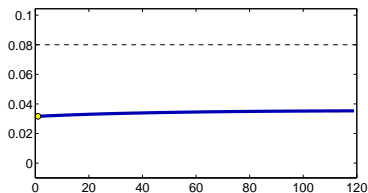
Capital tax



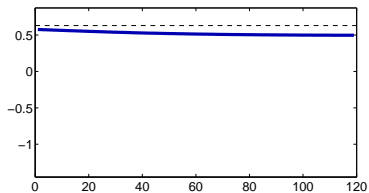
Labor tax



Lump sum to GDP



Debt to GDP

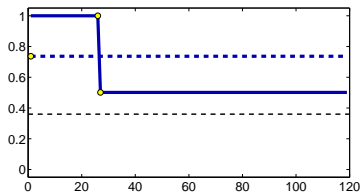


Number of Nodes **2**

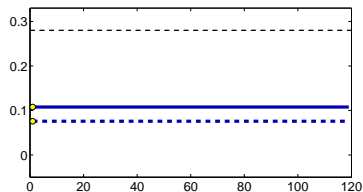
Avg. Welfare Gain 3.29

3 Choices

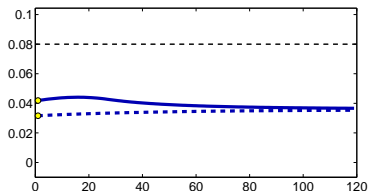
Capital tax



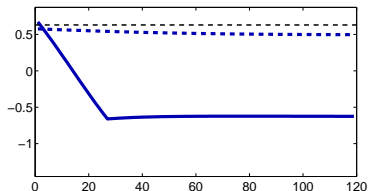
Labor tax



Lump sum to GDP



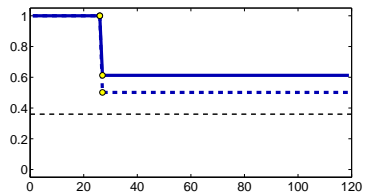
Debt to GDP



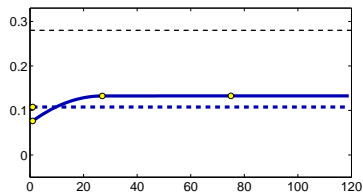
Number of Nodes	2	3
Avg. Welfare Gain	3.29	4.64

7 Choices

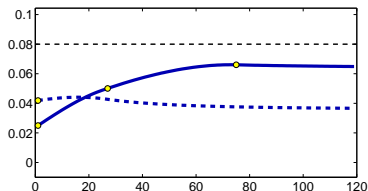
Capital tax



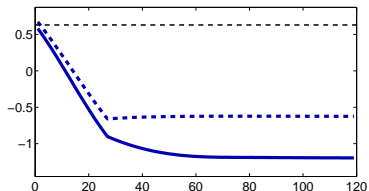
Labor tax



Lump sum to GDP



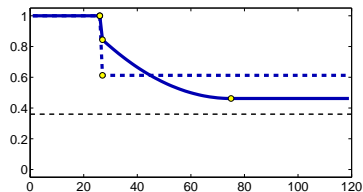
Debt to GDP



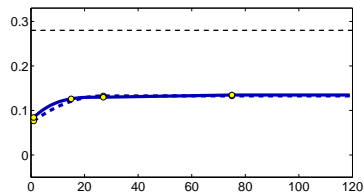
Number of Nodes	2	3	7
Avg. Welfare Gain	3.29	4.64	4.80

10 Choices

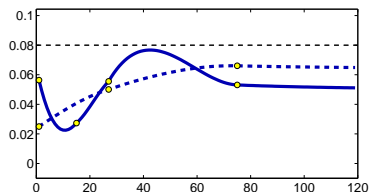
Capital tax



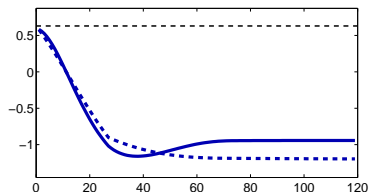
Labor tax



Lump sum to GDP



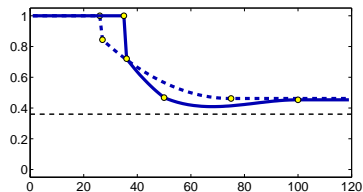
Debt to GDP



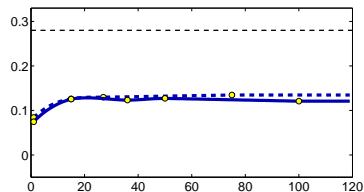
Number of Nodes	2	3	7	10
Avg. Welfare Gain	3.29	4.64	4.80	4.84

13 Choices

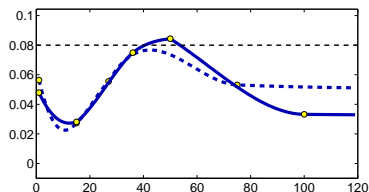
Capital tax



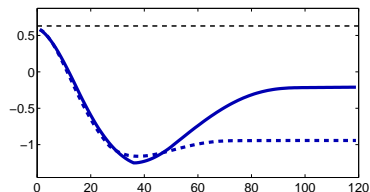
Labor tax



Lump sum to GDP



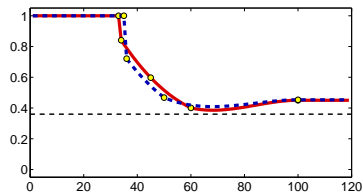
Debt to GDP



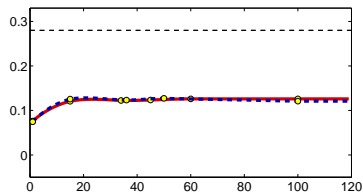
Number of Nodes	2	3	7	10	13
Avg. Welfare Gain	3.29	4.64	4.80	4.84	4.88

16 Choices

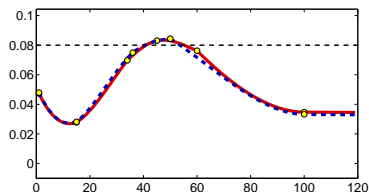
Capital tax



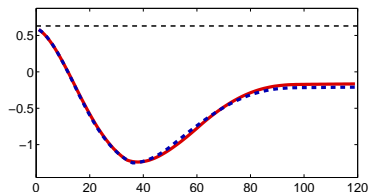
Labor tax



Lump sum to GDP



Debt to GDP



Number of Nodes	2	3	7	10	13	16
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Avg. Welfare Gain	3.29	4.64	4.80	4.84	4.88	4.90
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Long-run capital taxes

- The fact that capital taxes are positive in the long-run in the standard incomplete markets model has been established before. We are the first (to our knowledge) to quantify it and show that they are significant at 45%.
- Long-run capital taxes cannot be justified by the redistributive motive. The dependence of agents on initial conditions dissipates over time as they hit the borrowing constraint.
- Aiyagari (1995) and Chamley (2001) provide rationales.

Long-run capital taxes

- **Aiyagari (1995)** shows that, since there are no aggregate shocks, the planner's decision to move resources across time faces no risk whereas the individuals face idiosyncratic shocks and save for precautionary reasons.
- In order, for the intertemporal marginal rates of substitution of consumers and government to be equated a positive capital tax is required.
- This logic implies that the modified golden rule should hold in the long-run, our numerical results imply exactly that.
- **Chamley (2001)** shows that enough periods in the future every agent has the same probability of being in each of the possible individual states. It is, therefore, Pareto improving to redistribute in the long-run.

Role of the government debt

- The government accumulates assets over transition.
- Ricardian equivalence does not hold in the presence of the binding borrowing constraints.
- By accumulating assets, the government crowds in capital reducing interest rates and increasing wages.
- The associated changes in the composition of income are beneficial to the consumption-poor agents and reduces inequality.
- **Aiyagari & McGrattan (1998)**, solve for the level of debt that maximizes steady state welfare. Interestingly, they find that the optimal level is positive and very close to the current level (around 67%). **Röhrs & Winter (2011)** have argued that this is due to the underestimation of wealth inequality.

Transitory effects are important

	Labor tax τ^h	Capital tax τ^k	Transfers T/Y	Debt B/Y
Initial equilibrium	28.0	36.0	8.0	63.0
Constant optimal policy	7.6	73.7	3.5	49.8
Benchmark (long-run)	12.6	45.1	3.5	-16.9

- Optimal transition (Benchmark) vs. Constant optimal policy over transition: average welfare **gain of 1.9%**.

Other Long-Run Optimality Conditions

- Acikgoz (2013) and Hagedorn, Holter and Wang (2016) have made advancements towards characterizing the long-run optimal tax system in similar environments.
- They derive three long-run optimality conditions, including the modified golden rule, and propose an algorithm that allows for the computation of the optimal long-run tax system.

	τ^h	τ^k	T/Y	B/Y	K/Y	N
Benchmark	12.6	45.1	3.4	-16.9	2.82	0.39
Alternative Algorithm	15.5	46.3	5.3	-29.5	2.80	0.38

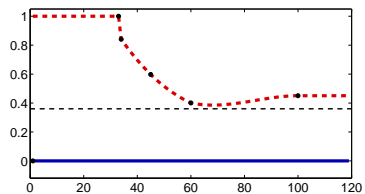
Role of the incomplete markets

Using an approach similar to [Werning \(2007\)](#), we characterize analytically the solution for the following simpler economies (with borrowing constraints substituted for No-Ponzi conditions):

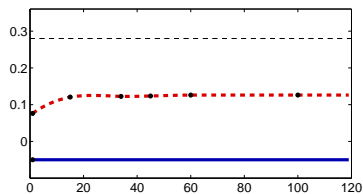
- Economy 1: Representative Agent ($\Gamma = I, e = 1, a_0 = \bar{a}$)
- Economy 2: Asset Heterogeneity ($\Gamma = I, e = 1$)
- Economy 3: Productivity Heterogeneity ($\Gamma = I, a_0 = \bar{a}$)
- Economy 4: Heterogeneity in Both ($\Gamma = I$)

Economy 1: representative agent

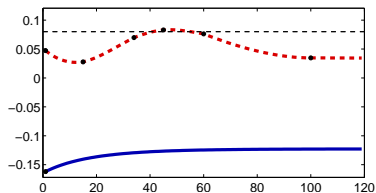
Capital tax



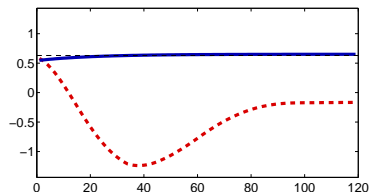
Labor tax



Lump sum to GDP

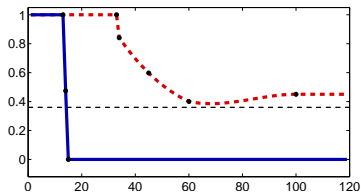


Debt to GDP

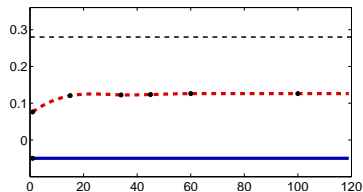


Economy 2: heterogeneity in initial assets

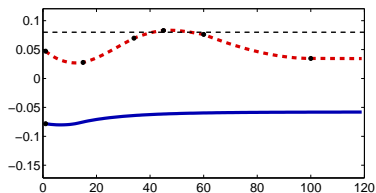
Capital tax



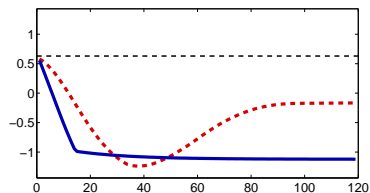
Labor tax



Lump sum to GDP

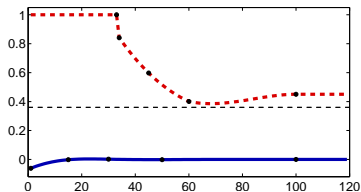


Debt to GDP

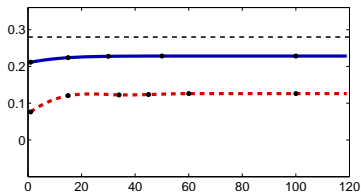


Economy 3: heterogeneity in productivity levels

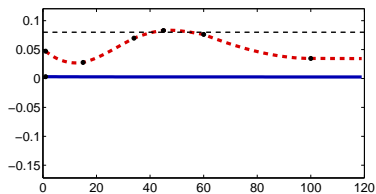
Capital tax



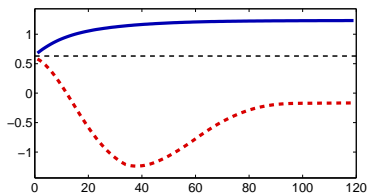
Labor tax



Lump sum to GDP

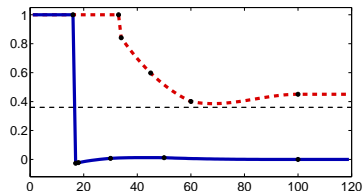


Debt to GDP

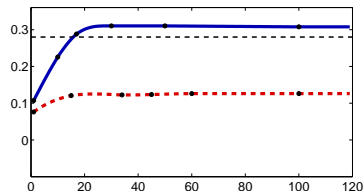


Economy 4: heterogeneity in both

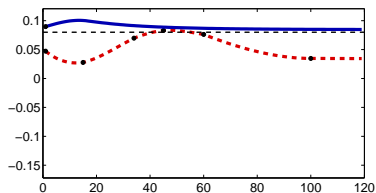
Capital tax



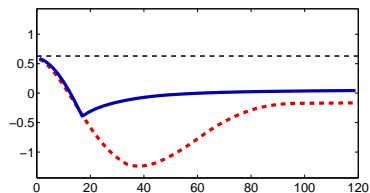
Labor tax



Lump sum to GDP



Debt to GDP



Economy 2: heterogeneity in initial assets

Proposition

There exists a finite integer $t^ \geq 1$ such that the optimal tax system is given by:*

$$\tau_t^k = \begin{cases} 1 & \text{for } 1 \leq t < t^* \\ \in [0, 1] & \text{for } t = t^* \\ 0 & \text{for } t > t^* \end{cases}$$

$$\tau_t^n = -\tau^c \quad \text{for } t \geq 1.$$

Economy 3: heterogeneity in productivity levels

Proposition

If capital taxes are bounded only by the positivity of gross interest rates, then

(i) τ_t^n is virtually constant at a level above $-\tau^c$;

(ii) it is optimal to set τ_t^k according to

$$\frac{1 + (1 - \tau_{t+1}^k)r_{t+1}}{1 + r_{t+1}} = \left(\frac{1 - \tau_{t+1}^n}{1 - \tau_t^n} \right) \left(\frac{\tau_t^n + \tau^c}{\tau_{t+1}^n + \tau^c} \right), \quad \text{for } t \geq 1.$$

Economy 4: heterogeneity in both

Proposition

There exists a finite integer $t^* \geq 1$ such that the optimal tax system is given by

- (i) $\tau_t^k = 1$ for $1 \leq t < t^*$;
- (ii) $\tau_{t^*}^k \in [0, 1]$;
- (iii) τ_t^k follows equation

$$\frac{1 + (1 - \tau_{t+1}^k)r_{t+1}}{1 + r_{t+1}} = \left(\frac{1 - \tau_{t+1}^n}{1 - \tau_t^n} \right) \left(\frac{\tau_t^n + \tau^c}{\tau_{t+1}^n + \tau^c} \right), \quad \text{for } t > t^*; \quad (1)$$

- (iv) for $1 \leq t < t^*$, τ_t^n is increasing and evolves according to (1);
- (v) for $t \geq t^*$, τ_t^n is virtually constant.

Related literature

- Aiyagari (1995), Chamley (2001): theoretical arguments for long-run capital taxes in our environment.
 - We are able to quantify the long-run capital taxes, and also the entire path of all of the fiscal instruments.
- Aiyagari & McGrattan (1998), Conesa, Kitao & Krueger (2009) solve a similar problem, but focus on steady state welfare.
 - We solve for the optimal time-varying policy over **transition**.
- Domeji & Heathcote (2004), Röhrs & Winters (2013): analysis of particular policies including welfare over transition.
 - We solve for the **optimal** policy.
- Davila, Hong, Krusell, Ríos-Rull (2012): characterize the constrained efficient allocation.
 - Increasing capital improves welfare through effect on prices.
 - $\text{Income} = (1 - \tau^k)ra + (1 - \tau^n)wn$

Related literature

- The New Dynamic Public Finance literature ([Golosov, Kocherlakota & Tsyvinski \(2003\)](#)).
 - Unrestricted instruments and design of mechanism to extract information about the agents' unobservable productivities.
 - Main results is the inverse Euler equation. [Farhi & Werning \(2012 JPE\)](#): welfare gains from its implementation are small.
 - Difficult to solve with persistent idiosyncratic shocks. [Farhi & Werning \(2012 REstud\)](#) and [Golosov, Troshkin & Tsyvinski \(2014\)](#) in partial equilibrium settings find that restrictions to linear taxes lead to small welfare losses.
 - Even if only as a benchmark to more elaborate tax systems, it is useful to understand the properties of a simpler optimal tax system in a quantitative general equilibrium environment.