Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks

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Motivation

- **How should a government set fiscal instruments over time to deal with inequality and individual risk?**
- Want to provide a quantitative answer to this question.
- Need a model that is able to replicate realistic levels of inequality and individual (uninsurable) risk.
- The standard incomplete markets model has been relatively successful in this front.
- In this environment, we formulate a Ramsey problem and solve numerically for the optimal transition with 4 instruments:
  - linear capital and labor income taxes, lump-sum transfers (or taxes), and government debt.

This paper

- We do not maximize steady state welfare. Instead, the solution maximizes welfare along the transition between an initial and a final steady state:
  - The initial steady state we calibrate to replicate key features of the US economy;
  - The final steady state is endogenously determined and depends on the (time-varying) paths of the fiscal instruments.
- Taxation **distorts** agents decisions, but also affects the composition of agent's income in ways that allow the planner to provide **redistribution** and **insurance**.
- The resolution of the associated trade-offs determine the optimal policy.

Findings

- Optimal levels of capital and labor income taxes are roughly consistent with the prevailing ones in the US; in the long run for a utilitarian planner and from the start for a planner that disregards equality concerns.
- High initial capital taxes are an efficient way to provide redistribution and are used in proportion to the planners degree of inequality aversion.
- It is possible to decompose the welfare gains from a proposed policy into what comes from reduction in distortions, insurance, and redistribution accounting for transitory effects.
- The welfare function is relatively flat with respect to movements in long-run fiscal instruments.
- Ignoring transition or the dynamics of taxes over time can be severely misleading.

Mechanism: Two-Period Economy

Why use distortive capital and labor income taxes when non-distortive lump-sum taxes are available?

Two-Period Economy - Uncertainty Economy

- Continuum of ex-ante identical agents receive ω in period 1.
- In period 2 agents have random productivity levels:

$$e_L = 1 - \frac{\varepsilon}{\pi}, \qquad e_H = 1 + \frac{\varepsilon}{1-\pi}.$$

- No insurance market: only risk-free asset, a, available.
- Agents solve

$$\max_{a, c_L, c_H, n_L, n_H} u\left(\omega - a, \bar{n}\right) + \beta\left[\pi u\left(c_L, n_L\right) + \left(1-\pi\right) u\left(c_H, n_H\right)\right]$$

$$\text{s.t.} \quad c_i = \left(1-\tau^n\right) w e_i n_i + (1 + (1-\tau^k) r) a + T, \quad i = L, H.$$

- In period 2, firms choose K and N to maximize profits given a CRS production function $f(K, N)$, and prices w and r.

Two-Period Economy

Definition

The Ramsey problem is to choose τ^k, τ^n, and T to maximize welfare (the expected utility of the agents) subject to the economy being in equilibrium.

Assumption (A)

No income effects on labor supply and constant Frisch elasticity, i.e.

$$u_{cn} - u_{cc}\frac{u_n}{u_c} = 0, \quad \text{and} \quad \frac{u_{cc}u_n}{n\left(u_{cc}u_{nn} - u_{cn}^2\right)} = \kappa.$$

- In a similar setup, Gottardi, Kajii and Nakajima (2014) characterize the solution to the Ramsey problem with Assumption A replaced by assumptions about the sign of general equilibrium effecs on prices.
- This assumption allows us to provide a sharper characterization.

Two-Period Economy - Uncertainty Economy

Proposition

In the uncertainty economy, if u satisfies Assumption A, then the optimal tax system is such that

$$\tau^n = \frac{(\nu - 1)\varepsilon}{(\nu - 1)\varepsilon + \kappa(\pi\nu - \pi + 1)} > 0, \text{ and}$$

$$\tau^k = 0,$$

where $\nu \equiv \frac{u_c(c_L, n_L)}{u_c(c_H, n_H)}$.

- **Insurance:** A positive labor income tax directly decreases the proportion of uncertain after tax labor income in total income.

Two-Period Economy - Inequality Economy

- Suppose now that productivity levels do not vary between agents, i.e. $e_L = e_H = 1$, but ω can take two values:

$$\omega_L = 1 - \frac{\epsilon}{p} \text{ (prop. } p), \;\; \omega_H = 1 + \frac{\epsilon}{1-p} \text{ (prop. } 1-p).$$

Proposition

In the inequality economy, if u satisfies Assumption A and has CARA or CRRA, then the optimal (utilitarian) tax system is such that

$$\tau^k = \frac{(\nu-1)\epsilon}{(\nu-1)\epsilon + \frac{1}{\psi}\rho(\tau^k, r)(\pi\nu - \pi + 1)} > 0, \textit{ and}$$

$$\tau^n = 0.$$

- **Redistribution:** A positive capital income tax directly decreases the proportion of unequal after tax capital income in total income.

Infinite-Horizon Model

Quantitative model in which we investigate the properties of the optimal policy

Environment - Households

- There is a measure one of households.
- Individual states: $a \in A$ - assets, and $e \in E$ - stochastic productivity that follows a Markov process with matrix Γ.
- Given a sequence of prices and taxes the household solves

$$v_t(a,e) = \max_{c_t, n_t, a_{t+1}} u(c_t, n_t) + \beta \sum_{e_{t+1} \in E} v_{t+1}(a_{t+1}, e_{t+1}) \Gamma_{e, e_{t+1}}$$

subject to

$$\begin{aligned} (1+\tau^c) c_t(a,e) + a_{t+1}(a,e) &= (1-\tau_t^n) w_t e n_t(a,e) + \\ &\quad + (1 + (1 - I_{\{a \geq 0\}} \tau_t^k) r_t) a + T_t \\ a_{t+1}(a,e) &\geq \underline{a}. \end{aligned}$$

Environment - Firm and Government

- Given prices, in each period, the representative firm solves

$$\max_{K_t, N_t} f(K_t, N_t) - w_t N_t - r_t K_t$$

- Government finances an exogenous stream of expenditure, and lump-sum transfers, with taxes on consumption, labor and capital, or debt

$$G + T_t + r_t B_t = B_{t+1} - B_t + \tau^c C_t + \tau_t^n w_t N_t + \tau_t^k r_t \hat{A}_t.$$

where $\hat{A}_t$ is the tax base for the capital income tax.

Equilibrium

Definition

Given K_0, B_0, an initial distribution λ_0 and a policy $\pi \equiv \{\tau_t^k, \tau_t^n, T_t\}_{t=0}^{\infty}$, a **competitive equilibrium** is a sequence of value functions $\{v_t\}_{t=0}^{\infty}$, an allocation $X \equiv \{c_t, n_t, a_{t+1}, K_{t+1}, N_t, B_{t+1}\}_{t=0}^{\infty}$, a price system $P \equiv \{r_t, w_t\}_{t=0}^{\infty}$, and a sequence of distributions $\{\lambda_t\}_{t=0}^{\infty}$, such that for all t:

1. Given P and π, $c_t(a,e)$, $n_t(a,e)$, and $a_{t+1}(a,e)$ solve the household's problem and $v_t(a,e)$ is the respective value function;
2. Factor prices are set competitively: $r_t = f_K(K_t, N_t)$, $w_t = f_N(K_t, N_t)$;
3. The probability measure λ_t is consistent with Γ and $a_{t+1}(a,e)$;
4. Government budget constraint holds and debt is bounded;
5. Markets clear,

$$C_t + G + K_{t+1} - K_t = f(K_t, N_t), \quad K_t + B_t = \int_{A\times E} a_t(a,e)d\lambda_t.$$

Procedure

1. We calibrate the initial stationary equilibrium to match data on macro aggregates, wealth, income and earnings inequality, statistics about the labor income process, and the current levels of the fiscal instruments.

2. Then, given this and paths for the fiscal instruments we can compute a transition to an endogenously determined final steady state.

3. Finally, we parametrize these paths and optimize in the space of sequences of fiscal instruments.

Calibration

Calibration: Preferences and Technology

- Preferences and technology:

$$u(c,n) = \frac{1}{1-\sigma}\left(c - \chi \frac{n^{1+\frac{1}{\kappa}}}{1+\frac{1}{\kappa}}\right)^{1-\sigma}, \quad f(K,N) = K^{\alpha} N^{1-\alpha} - \delta K$$

Statistic	Target	Model	Parameter	Value
Preferences and Technology				
Intertemporal elast. of subst.	-	0.50	σ	2.00
Frisch elasticity	-	0.72	κ	0.72
Average hours worked	0.30	0.30	χ	3.91
Capital to output	2.72	2.71	β	0.95
Capital income share	-	0.38	α	0.38
Investment to output	0.27	0.27	δ	0.10

Calibration: Fiscal Policy

Statistic	Target	Model	Parameter	Value
Borrowing Constraint				
Hh with negative wealth (%)	18.6	19.3	$\underline{a}/Y$	-0.025
Fiscal Policy				
Capital income tax (%)	-	36.0	τ^k	0.36
Labor income tax (%)	-	28.0	τ^n	0.28
Consumption tax (%)	-	5.0	τ^c	0.05
Transfer to output (%)	8.0	8.0	T/Y	0.08
Debt to Output (%)	63.0	63.0	G/Y	0.15

- In order to set the tax rates in the initial steady state, we use the effective average tax rates (tax revenue over tax base for each source) computed by Trabandt and Uhlig (2011) for 1995-2007.

Calibration: Distributions

	Quintiles							
	0-5%	1st	2nd	3rd	4th	5th	95-100%	Gini
	Wealth Distribution							
Target	−0.2	−0.2	1.1	4.5	11.2	83.4	60.3	0.82
Model	−0.0	−0.1	0.3	2.1	10.8	86.9	58.4	0.84
	Earnings Distribution							
Target	−0.1	−0.1	4.2	11.7	20.8	63.5	35.3	0.64
Model	0.5	3.4	4.1	8.3	19.7	64.5	34.5	0.61
	Income Distribution							
Target	0.2	2.8	6.7	11.3	18.3	60.9	36.9	0.57
Model	0.9	4.6	5.8	9.6	22.6	57.5	31.2	0.55

Notes: Data come from the 2007 Survey of the Consumer Finance.

Calibration: Labor Income Process

Statistic	Target	Model
Statistical Properties of Labor Income Process		
Variance of 1-year diff.	0.26	0.27
Skewness of 1-year diff.	-1.07	-0.75
Kurtosis of 1-year diff.	14.93	14.58
Variance of 5-year diff.	0.61	0.64
Skewness of 5-year diff.	-1.25	-0.82
Kurtosis of 5-year diff.	9.51	10.19
Autocorrelation	0.88	0.88

Notes: Estimates come from Guvenen, Karahan, Ozkan & Song (2015).

Calibration: Labor Income Process Parameters

Model Parameters
Persistent Shock
$\Gamma_P = \begin{bmatrix} 0.96 & 0.04 & 0.00 & 0.00 \\ 0.08 & 0.91 & 0.01 & 0.00 \\ 0.01 & 0.00 & 0.98 & 0.01 \\ 0.09 & 0.01 & 0.01 & 0.89 \end{bmatrix} \quad e_P = \begin{bmatrix} 0.48 \\ 0.88 \\ 1.80 \\ 7.14 \end{bmatrix}$
Transitory Shock
$P_T = \begin{bmatrix} 0.05 \\ 0.04 \\ 0.12 \\ 0.59 \\ 0.10 \\ 0.11 \end{bmatrix} \quad e_T = \begin{bmatrix} -0.23 \\ -0.10 \\ -0.06 \\ 0.01 \\ 0.18 \\ 0.21 \end{bmatrix}$

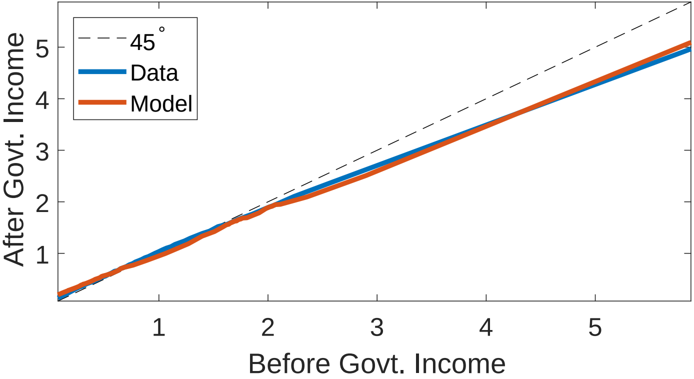
Model Performance

- Without targeting we approximate well the composition of income by quintiles of income:

Quintile	US Data			Model		
	Labor	Asset	Transfer	Labor	Asset	Transfer
1st	38.4	-1.9	63.5	57.0	0.7	42.3
2nd	66.4	2.5	31.1	60.8	5.9	33.3
3rd	78.6	2.7	18.7	73.9	6.1	20.0
4th	85.4	4.0	10.6	71.0	20.4	8.5
5th	77.5	18.2	4.3	78.4	18.3	3.3
All	77.3	12.2	10.4	74.3	16.1	9.6

Model Performance: Income tax schedule

- The tax rates are calibrated to match effective tax rates. However, we also approximate well the actual income tax schedule (data from Heathcote, Storesletten & Violante (2014)).



Notes: The axis units are income relative to the mean.

Ramsey Problem

Ramsey Problem

Definition

Given λ_0, K_0, B_0, τ_0^k, τ_0^n, T_0 and a welfare function W, the **Ramsey problem** is $\max_\pi W(X(\pi))$ subject to $X(\pi)$ being an equilibrium allocation and π satisfying $\tau_t^k \leq 1 \ \forall t \geq 1$.

- The benchmark welfare function is utilitarian:

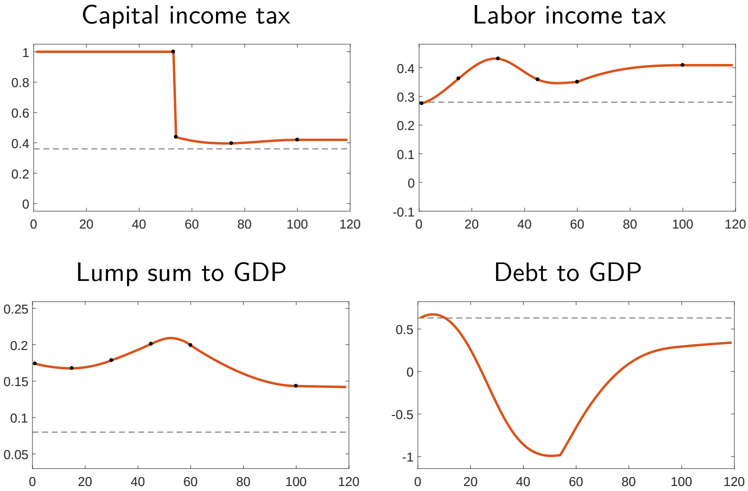
$$W(\pi) = \int_S E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\left(a, e|\pi\right), n_t\left(a, e|\pi\right)\right) d\lambda_0.$$

- Solving this problem involves searching on the space of sequences $\{\tau_t^k, \tau_t^n, T_t\}_{t=1}^{\infty}$.

- In order to make it computationally feasible we approximate these sequences with cubic splines with nodes at prespecified periods.

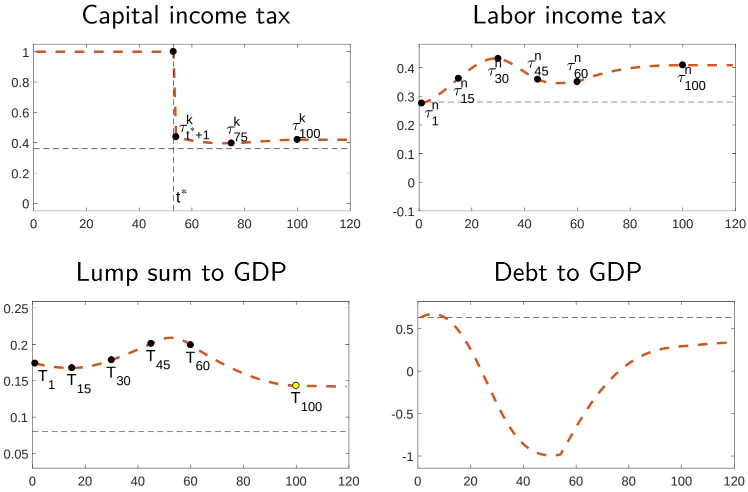
Results

Main result



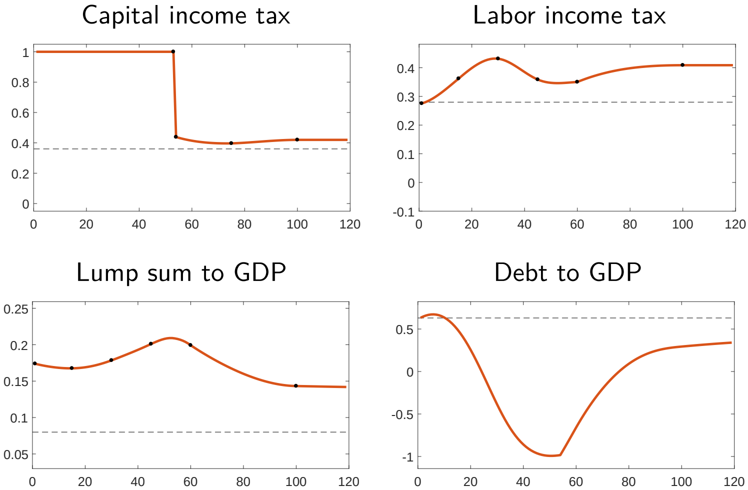
Enough Nodes   Aggregates

Main result



Enough Nodes Aggregates

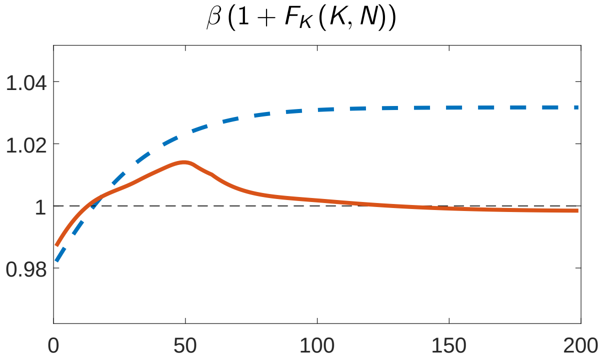
Main result



- Average welfare gain equivalent to a permanent 13.86% increase in consumption.

Enough Nodes Aggregates

Modified golden rule holds in the final steady state



Red line: Benchmark; Blue line: Economy with constant optimal policy over transition.

Other Long-run Optimality Conditions

Main result

- Conditional on the initial type (a, e), what percentage of the population would find the reform beneficial?

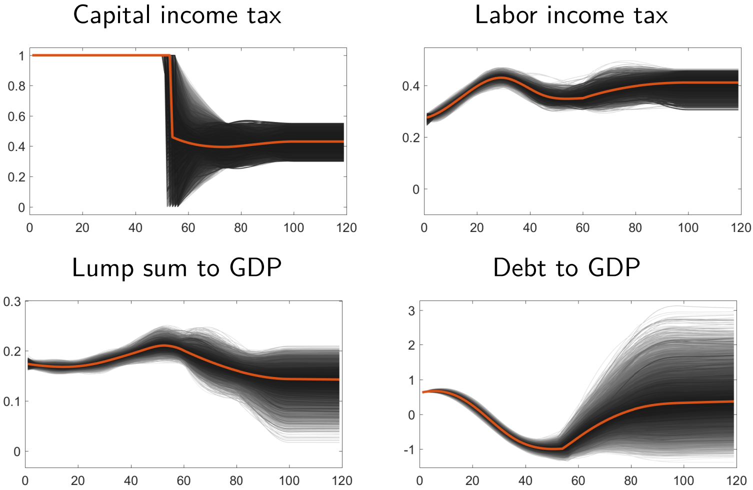
Quintile	$e = 0.48$	$e = 0.88$	$e = 1.80$	$e = 7.14$	All
1st	100.0	100.0	0.0	0.0	99.8
2nd	100.0	100.0	0.0	0.0	99.1
3rd	100.0	100.0	0.0	0.0	89.6
4th	100.0	100.0	0.0	0.0	27.8
5th	18.0	13.5	0.0	0.0	9.1
All	88.1	88.0	0.0	0.0	65.3

- Income composition changes: share of labor and asset income decrease. Income composition dynamics

Flatness of Welfare

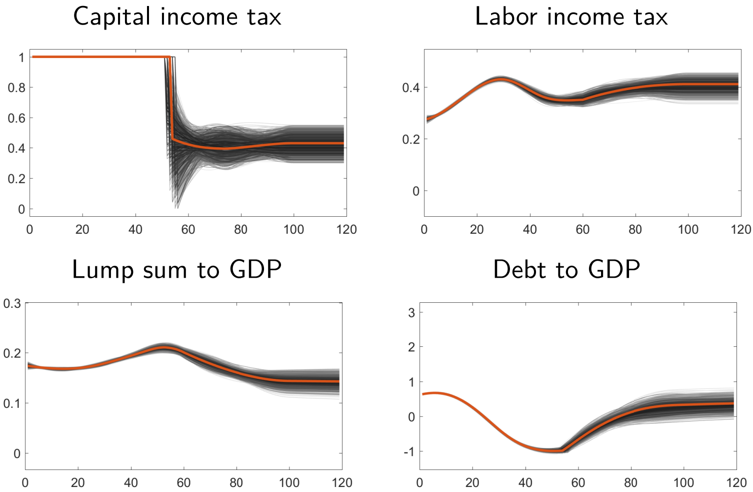
The welfare function is relatively flat with respect to movements in long-run fiscal instruments

Flatness of Welfare



- All paths plotted in this figure are associated with a welfare gain higher than 13.76% - less than 0.1% away from the optimal path.

Flatness of Welfare



- All paths plotted in this figure are associated with a welfare gain higher than 13.85% - less than 0.01% away from the optimal path.

Welfare Decomposition

We can decompose the welfare gains into what comes from the reduction in distortions, redistribution, and insurance

Welfare decomposition

Details

- The utilitarian welfare function can increase for three reasons:

  1. Reduction in distortions, if the utility of the average agent, $\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$, increases: **the level effect (Δ_L)**;
  2. Transfers from ex-post rich to ex-post poor, if the uncertainty of each individual path $\{c_t, n_t\}_{t=1}^{\infty}$ is reduced: **the insurance effect (Δ_I)**;
  3. Transfers from ex-ante rich to ex-ante poor, if the inequality between certainty equivalents for $\{c_t, n_t\}_{t=1}^{\infty}$ is reduced: **the redistribution effect (Δ_R)**.

Proposition

Let Δ be the utilitarian (average) welfare gain. The following decomposition holds:

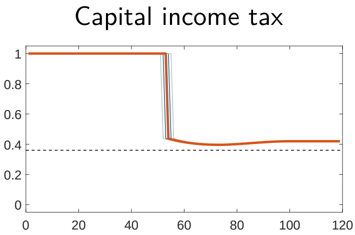
$$(1+\Delta) = (1+\Delta_L)(1+\Delta_I)(1+\Delta_R)$$

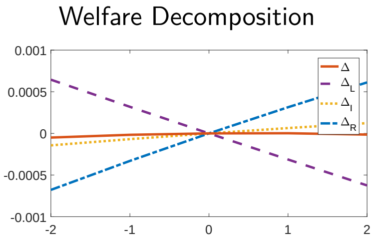
Δ	Δ_L	Δ_I	Δ_R
13.9	-4.8	2.4	16.8

Variations Around the Optimal Taxes

Variations Around the Optimal Taxes

Number of years of capital income taxes in the upper bound - t^*



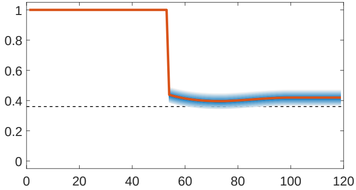


- The effect on insurance is of second order; the relevant trade-off is between redistribution and the level effect.
- These effects, however, almost exactly offset leading to a relatively flat average welfare function.

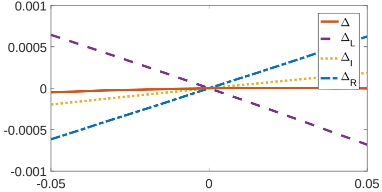
Variations Around the Optimal Taxes

Long-run capital income taxes

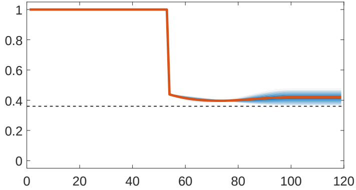
Capital income tax



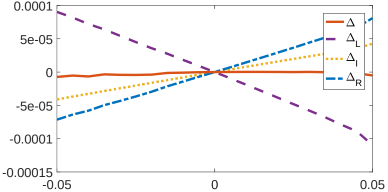
Welfare Decomposition



Capital income tax



Welfare Decomposition



Variations Around the Optimal Taxes

Long-run capital income taxes

- Aiyagari (1995): since agents face idiosyncratic shocks and save for precautionary reasons, for the intertemporal marginal rates of substitution of consumers and government to be equated a positive capital tax is required (modified golden rule follows).[1]

- Chamley (2001): enough periods in the future every agent has the same probability of being in each of the possible individual states. It is, therefore, Pareto improving to redistribute in the long-run.

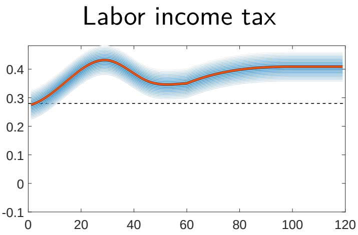
- Even focusing only of the productivity states we have that $\max_{i,j}\left|\left(\Gamma^{100}\right)_{i,j}-\pi_j\right|\approx 0.1$. So, for Chamley (2001)'s logic to kick in we would need to consider changes in taxes after 100 years.

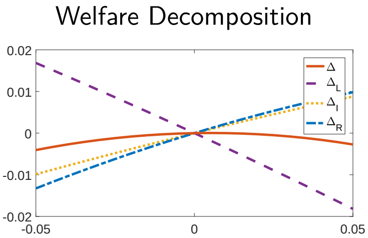
- Notice precision at which we need to compute welfare gains.

[1] This argument assumes a Ramsey steady state as we have done.

Variations Around the Optimal Taxes

Labor income taxes



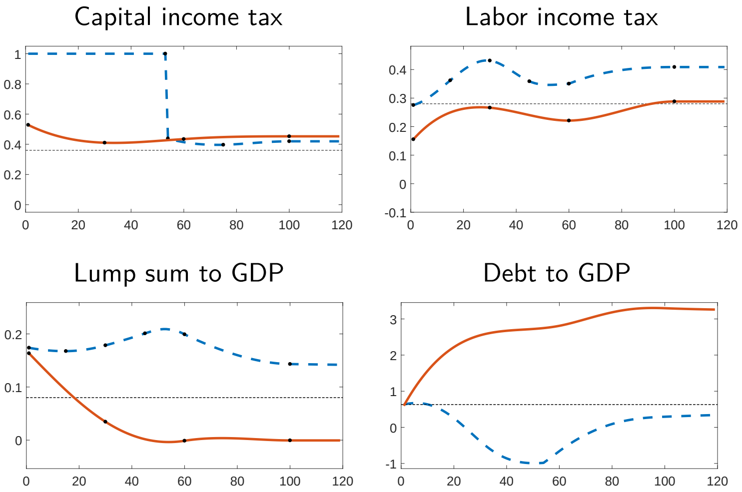


- The effect of changes in average labor income taxes are an order of magnitude higher than the changes to capital income taxes considered above.
- The insurance effect plays a comparable role to the redistribution effect in determining the optimal level of labor income taxes.

Maximizing Efficiency

Maximizing level and insurance effects: $(1+\Delta_L)(1+\Delta_I)$

Maximizing Efficiency



- Welfare gain equivalent to a permanent 3.4% increase in consumption ($\Delta_L = 1.2\%$ and $\Delta_I = 2.2\%$)

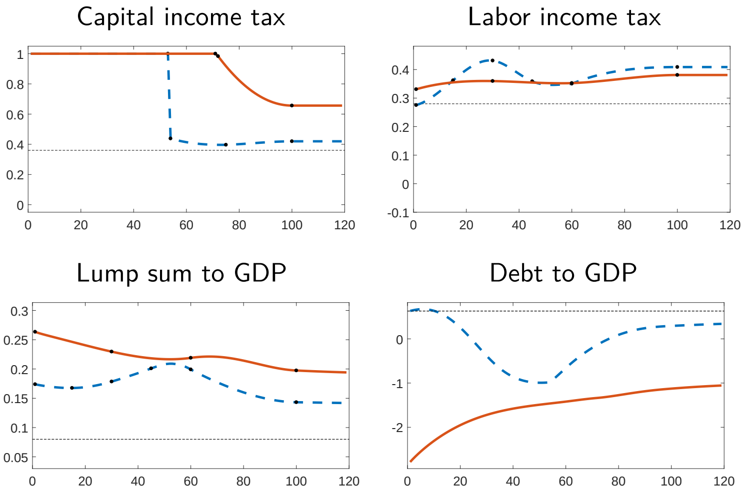
Maximizing Efficiency

- Long-run capital income taxes are remarkably similar to the benchmark one. No redistribution via high initial taxes though.
- Labor income taxes are about 10% lower throughout the transition. The only benefit to welfare is via insurance.
- Lower overall taxes on capital and labor income implies lower overall lump-sum transfers.
- Lump-sum transfer front-loaded to relax borrowing constraints.
- This, however, has the negative side effect of increasing government debt, which crowds out capital.
- Lump-sum transfers are not front-loaded in the benchmark experiment, because this effect in combination with the high initial capital income taxes would lead to too much crowding out.

Capital Levy

What if we allow taxes on capital income in period 0?

Capital Levy



- Welfare gain equivalent to a permanent 32.4% increase in consumption ($\Delta_L = -2.8\%$, $\Delta_I = 1.9\%$, and $\Delta_R = 33.7\%$)

Capital Levy

- 99 percent of assets holdings is expropriated. Surprisingly, this increases optimal capital income taxes in the future.
- Savings becomes even more inelastic as high productivity agents want to build back their precautionary savings buffer, therefore capital taxes are less distortive.
- The huge accumulation of assets by the government in period 0 actually leads to an initial crowding in of capital.
- On the other hand, capital taxes are still useful to provide redistribution and insurance.

Transitory effects

Ignoring transitory effects or dynamics of taxes over time can be severely misleading

Transitory effects are important

	Capital tax τ^k	Labor tax τ^n	Transfers T/Y	Debt B/Y
Initial equilibrium	36.0	28.0	8.0	63.0
Ignoring transition	–	43.4	16.1	−375.2
Benchmark (long-run)	42.0	40.9	13.9	40.0

- If transition is ignored in particular the cost associated with reducing government debt and accumulating capital are not taken under consideration.
- Then, the planner chooses a debt-to-output ratio of −375.2%. At this level the amount of capital that is crowded in is close to the golden rule level, that is, such that interest rates (net of depreciation) equal to zero.

More on Government Debt

Transitory effects are important

	Capital tax τ^k	Labor tax τ^n	Transfers T/Y	Debt B/Y
Initial equilibrium	36.0	28.0	8.0	63.0
Constant policy	96.1	34.9	20.6	−95.5
Benchmark (long-run)	42.0	40.9	13.9	40.0

- Since the government cannot change taxes over time and the welfare function puts higher weight on the short run, the optimal taxes are close to the ones in the short run of the optimal dynamic taxes. Constant Policy

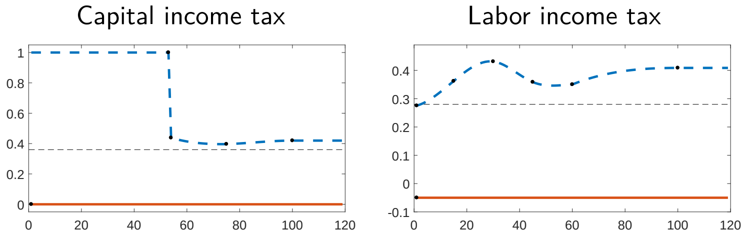
Role of market incompleteness

Role of market incompleteness

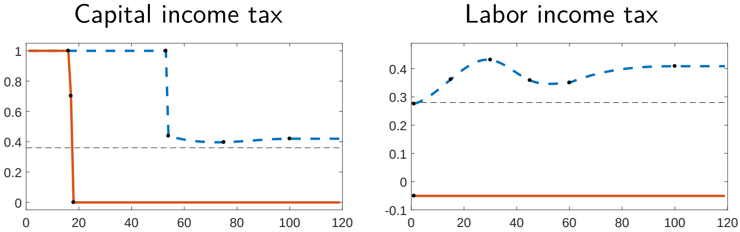
Using an approach similar to Werning (2007), we characterize analytically the solution for the following simpler economies (with borrowing constraints substituted for No-Ponzi conditions):

- Economy 1: Representative Agent ($\Gamma = I$, $e = 1$, $a_0 = \bar{a}$)
- Economy 2: Asset Heterogeneity ($\Gamma = I$, $e = 1$)
- Economy 3: Productivity Heterogeneity ($\Gamma = I$, $a_0 = \bar{a}$)
- Economy 4: Heterogeneity in Both ($\Gamma = I$)

Economy 1: representative agent

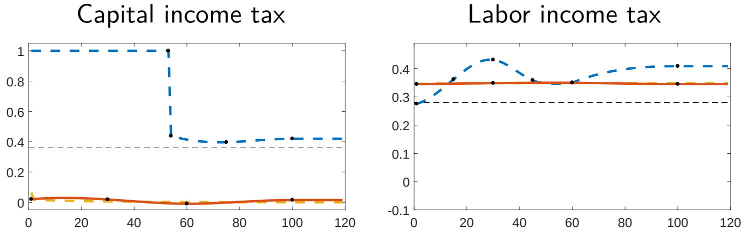


Economy 2: heterogeneity in initial assets



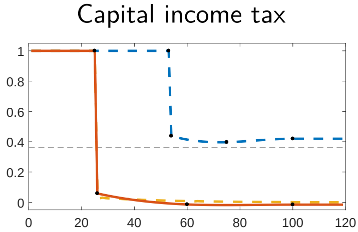
Proposition

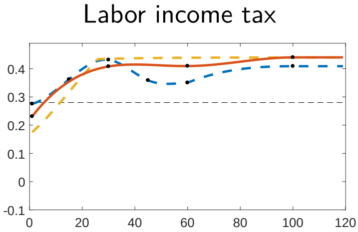
Economy 3: heterogeneity in productivity levels



Proposition

Economy 4: heterogeneity in both



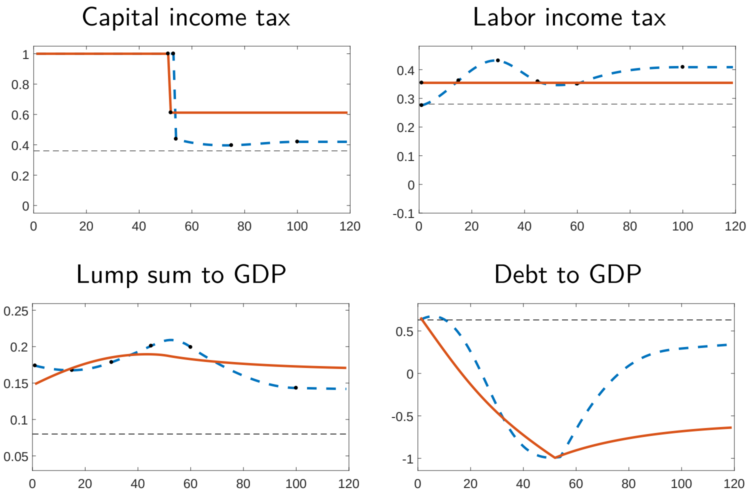


Proposition

Inequality Aversion

Alternative welfare functions

A good approximation



- The solution with 3 choices produces a reasonable approximation for the benchmark solution. Welfare gains: 13.19 vs. 13.87.

Welfare functions

- The utilitarian welfare function implies a particular social preference with respect to the equality versus efficiency trade-off.
- To rationalize different preferences about this trade-off, we use the following welfare function[2]

$$W^{\hat{\sigma}} = \left(\int \bar{x}_0 \left(a_0, e_0 \right)^{1-\hat{\sigma}} d\lambda_0 \right)^{\frac{1}{1-\hat{\sigma}}}$$

- If $\hat{\sigma} = \sigma$, $W^{\hat{\sigma}}$ is equivalent to the utilitarian welfare function.
- If $\hat{\sigma} = 0$, maximizing $W^{\hat{\sigma}}$ is akin to maximizing $\left(1 + \Delta_L\right)\left(1 + \Delta_I\right)$, i.e. the planner has no equality concerns.
- As $\hat{\sigma} \to \infty$, $W^{\hat{\sigma}}$ approaches $\min \left(\bar{x}_0 \left(a_0, e_0 \right) \right)$.

[2] $\bar{x}_0$ denotes the certainty equivalent of a consumption-labor composite at $t = 0$.

Different degrees of inequality aversion

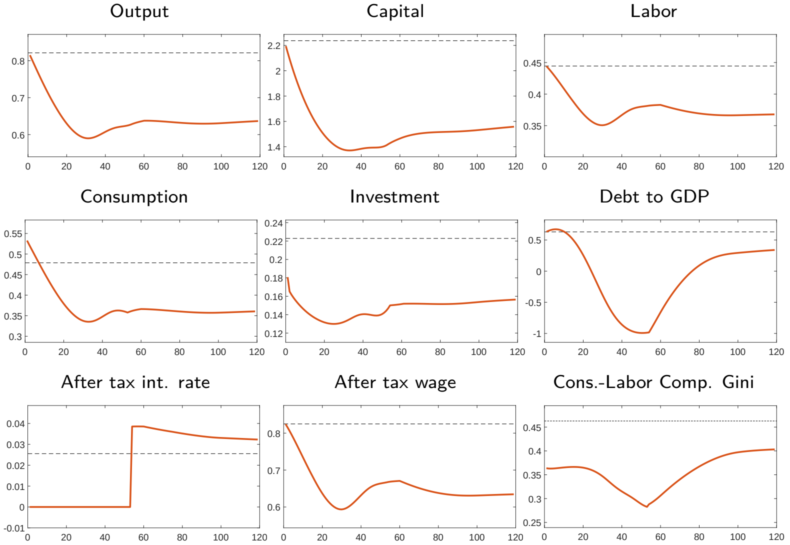
- Higher $\hat{\sigma}$ imply higher t^*'s. However, the final levels of capital and labor taxes are remarkably unaffected by changes in this parameter.

	t^*	τ^k	τ^n	T/Y	B/Y
$\hat{\sigma} = 0.0$	0	53.0	27.5	10.2	40.3
$\hat{\sigma} = 1.0$	35	69.3	31.0	15.6	-45.4
$\hat{\sigma} = 2.0^*$	51	61.2	35.4	16.8	-59.4
$\hat{\sigma} = 3.0$	53	61.2	37.5	17.8	-65.4
$\hat{\sigma} = 4.0$	54	61.5	38.5	18.4	-68.9
$\hat{\sigma} = 5.0$	55	61.5	39.2	18.6	-70.9

Conclusions

- Numerically, we are able to approximate the solution of the Ramsey problem in the standard incomplete markets model.
- We find that optimal levels of capital and labor income taxes are roughly consistent with the prevailing ones in the US.
- High initial capital taxes are an efficient way to provide redistribution and are used in proportion to the planners degree of inequality aversion.
- We introduce a welfare decomposition method that is able to account for transitory effects.
- The welfare function is relatively flat with respect to movements in long-run fiscal instruments.
- The solution method can be applied to different models: introducing human capital or life cycle, for instance, would only be harder to the extent that it takes longer to compute the transition.

Aggregates



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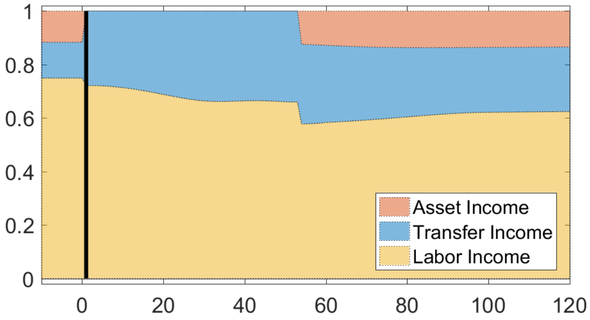
Income composition dynamics



Notes: From top to bottom the areas represent the shares of asset, transfer and labor income; before time 0 the areas represent the shares in the initial stationary equilibrium.

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Welfare Decomposition

- Average welfare gain, Δ:

$$\int E_0 \left[U\left(\left\{ x_t^R \right\} \right) \right] d\lambda_0 \left(a_0, e_0 \right) = \int E_0 \left[U\left(\left(1 + \Delta \right) \left\{ x_t^{NR} \right\} \right) \right] d\lambda_0 \left(a_0, e_0 \right),$$

where λ_0 is the initial distribution of initial states (a_0, e_0).

- Aggregate consumption-labor composite, X_t^j:

$$X_t^j \equiv \int x_t^j \left(a_0, e^t \right) d\lambda_t^j \left(a_0, e^t \right), \quad \text{for } j = R, NR.$$

- Individual certainty equivalent, $\bar{x}_t^j \left(a_0, e_0 \right) \equiv \eta^j \left(a_0, e_0 \right) x_t^j \left(a_0, e_0 \right)$:

$$U\left(\left\{ \bar{x}_t^j \right\} \right) \equiv E_0 \left[U\left(\left\{ x_t^j \right\} \right) \right], \quad \text{for } j = R, NR.$$

- Aggregate certainty equivalent, $\bar{X}^j$:

$$\bar{X}_t^j \equiv \int \bar{x}_t^j \left(a_0, e_0 \right) d\lambda_0 \left(a_0, e_0 \right), \quad \text{for } j = R, NR.$$

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Welfare Decomposition

- Level Effect, Δ_L:

$$U\left(\left\{X_t^R\right\}\right) = U\left(\left(1+\Delta_L\right)\left\{X_t^{NR}\right\}\right).$$

- Insurance Effect, Δ_I:

$$U\left(\left(1-p_{unc}^j\right)\left\{X_t^j\right\}\right) = U\left(\left\{\bar{X}_t^j\right\}\right), \text{ then,}$$

$$1+\Delta_I = \frac{1-p_{unc}^R}{1-p_{unc}^{NR}}.$$

- Redistribution Effect, Δ_R:

$$U\left(\left(1-p_{ine}^j\right)\left\{\bar{X}_t^j\right\}\right) = \int U\left(\left\{\bar{x}_t^j\left(a_0, e_0\right)\right\}\right) d\lambda\left(a_0, e_0\right), \text{ then,}$$

$$1+\Delta_R = \frac{1-p_{ine}^R}{1-p_{ine}^{NR}}.$$

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Other Long-Run Optimality Conditions

- Acikgoz, Hagedorn, Holter and Wang (2018) have made advancements towards characterizating the long-run optimal tax system in similar environments.

- They derive three long-run optimality conditions, including the modified golden rule, and propose an algorithm that allows for the computation of the optimal long-run tax system.[3]

	τ^k	τ^n	T/Y	B/Y	K/Y	N
Benchmark	45.1	12.6	3.4	-16.9	2.82	0.39
Alternative Algoritim	46.3	15.5	5.3	-29.5	2.80	0.38

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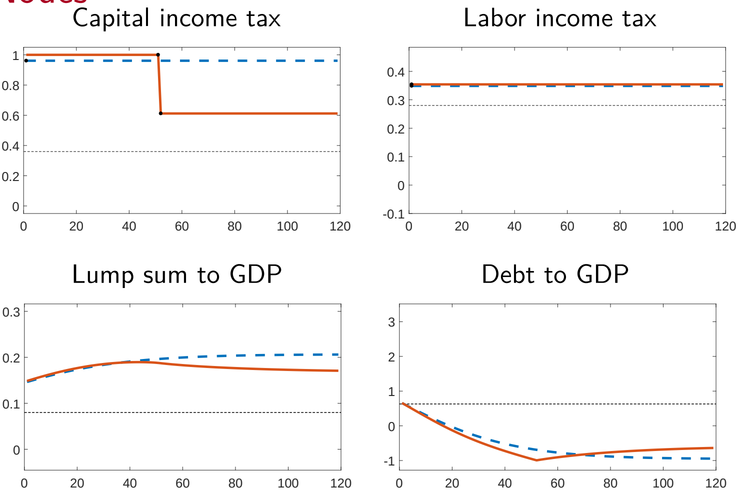
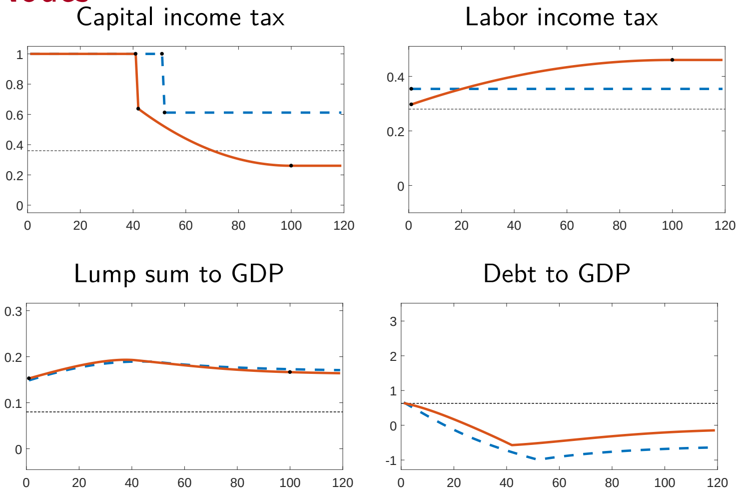
[3]These results were obtained under a previous calibration of the model.

2→3 Nodes



Nodes	2	3
Welfare	12.65	13.19

3→6 Nodes



Nodes	2	3	6
Welfare	12.65	13.19	13.46

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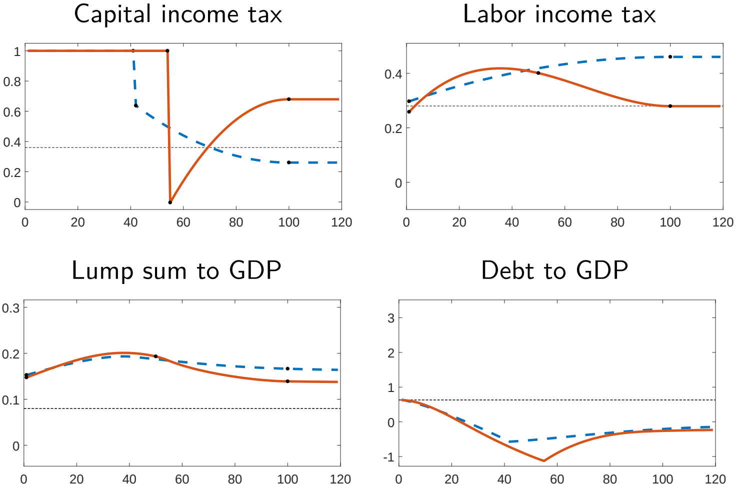
6→8 Nodes



Nodes	2	3	6	8
Welfare	12.65	13.19	13.46	13.57

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8→10 Nodes

Nodes	2	3	6	8	10
Welfare	12.65	13.19	13.46	13.57	13.83

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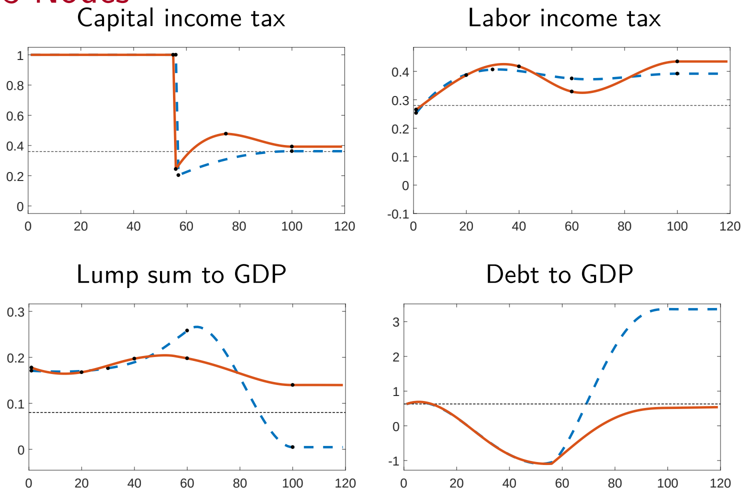
10→13 Nodes



Nodes	2	3	6	8	10	13
Welfare	12.65	13.19	13.46	13.57	13.83	13.86

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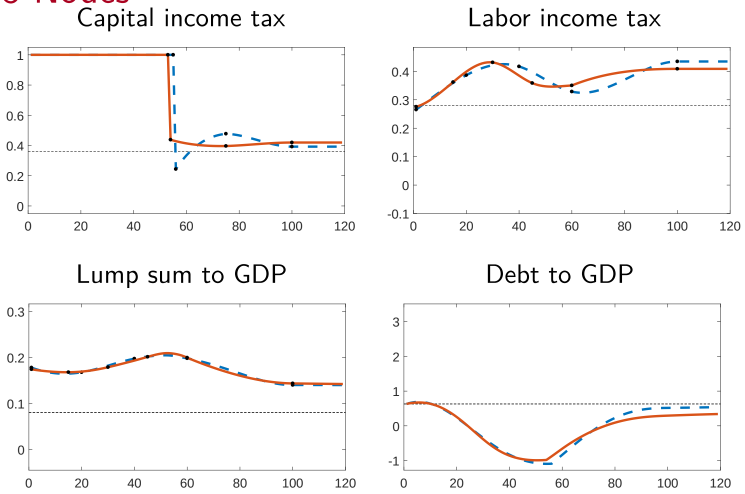
13→15 Nodes



Nodes	2	3	6	8	10	13	15
Welfare	12.65	13.19	13.46	13.57	13.83	13.86	13.87

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Role of the government debt

- Ricardian equivalence does not hold in the presence of the binding borrowing constraints.
- By accumulating assets, the government crowds in capital reducing interest rates and increasing wages.
- The associated changes in the composition of income are beneficial for the level and redistribution effects but detrimental to the insurance effect.
- Aiyagari & McGrattan (1998), solve for the level of debt that maximizes steady state welfare. Interestingly, they find that the optimal level is positive and very close to the current level (around 67%). Röhrs & Winter (2011) have argued that this is due to the underestimation of wealth inequality.

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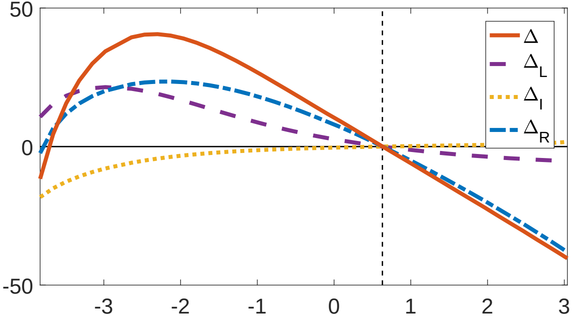
Role of the government debt

Welfare decomposition versus debt-to-gdp in steady state



Notes: the variable in the *x*-axis is the debt-to-gdp in steady state; the thin dashed vertical line marks the level of debt-to-gdp in the initial stationary equilibrium, 63%, versus which the welfare changes are calculated. As in Aiyagari & McGrattan (1998) a balanced budget is guaranteed by a total income tax.

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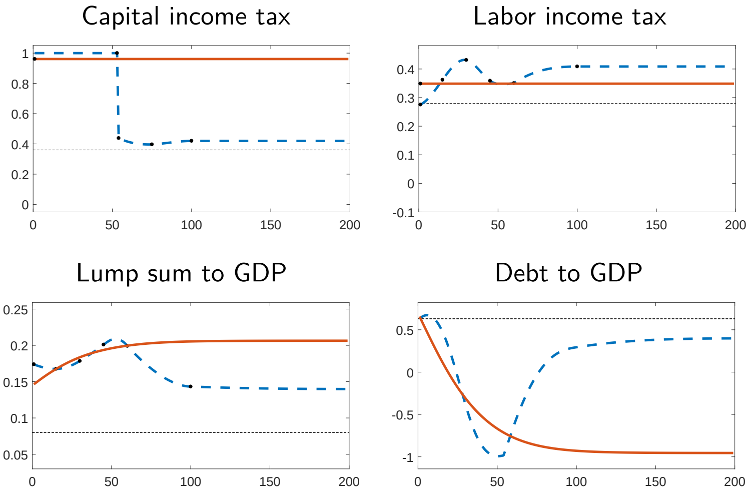
Constant policy



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Economy 2: heterogeneity in initial assets

Proposition

There exists a finite integer $t^* \geq 1$ such that the optimal tax system is given by:

$$\tau_t^k = \begin{cases} 1 & \text{for } 1 \leq t < t^* \\ \in [0,1] & \text{for } t = t^* \\ 0 & \text{for } t > t^* \end{cases}$$

$$\tau_t^n = -\tau^c \quad \text{for } t \geq 1.$$

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Economy 3: heterogeneity in productivity levels

Proposition

If capital taxes are bounded only by the positivity of gross interest rates, then

(i) τ_t^n *is virtually constant at a level above* $-\tau^c$*;*

(ii) *it is optimal to set* τ_t^k *according to*

$$\frac{1+(1-\tau_{t+1}^k)r_{t+1}}{1+r_{t+1}} = \left(\frac{1-\tau_{t+1}^n}{1-\tau_t^n}\right)\left(\frac{\tau_t^n+\tau^c}{\tau_{t+1}^n+\tau^c}\right), \quad \text{for } t \geq 1.$$

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Economy 4: heterogeneity in both

Proposition

There exists a finite integer $t^* \geq 1$ such that the optimal tax system is given by

(i) $\tau_t^k = 1$ *for* $1 \leq t < t^*$*;*

(ii) $\tau_{t^*}^k \in [0, 1]$*;*

(iii) τ_t^k *follows equation*

$$\frac{1 + (1 - \tau_{t+1}^k) r_{t+1}}{1 + r_{t+1}} = \left(\frac{1 - \tau_{t+1}^n}{1 - \tau_t^n} \right) \left(\frac{\tau_t^n + \tau^c}{\tau_{t+1}^n + \tau^c} \right), \quad \text{for } t > t^*; \quad (1)$$

(iv) *for* $1 \leq t < t^*$*,* τ_t^n *is increasing and evolves according to (1);*

(v) *for* $t \geq t^*$*,* τ_t^n *is virtually constant.*

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Related literature

- Ramsey with complete markets (Chamley (1986), Judd (1985)).
  - In our model agents face uninsurable risk. The main qualitative difference about the solution is that capital taxes should be positive in the long-run.
- Heathcote, Storesletten & Violante (2014), Gottardi, Kajii & Nakajima (2014): elegant analytical solutions in similar, though more stylized, environments.
  - Our approach is more quantitative. We lose analytical tractability, but can solve the problem numerically.

Related literature

- Aiyagari (1995), Chamley (2001): theoretical arguments for long-run capital taxes in our environment.
  - We are able to quantify the long-run capital taxes, and also the entire path of all of the fiscal instruments.
- Aiyagari & McGrattan (1998), Conesa, Kitao & Krueger (2009) solve a similar problem, but focus on steady state welfare.
  - We solve for the optimal time-varying policy over **transition**.
- Domeji & Heathcote (2004), Röhrs & Winters (2013): analysis of particular policies including welfare over transition.
  - We solve for the **optimal** policy.
- Davila, Hong, Krusell, Ríos-Rull (2012): characterize the constrained efficient allocation.
  - Pecuniary externalities. Income $= (1-\tau^k)ra + (1-\tau^n)wn$.

Related literature

- The New Dynamic Public Finance literature (Golosov, Kocherlakota & Tsyvinski (2003)).
  - Unrestricted instruments and design of mechanism to extract information about the agents' unobservable productivities.
  - Main results is the inverse Euler equation. Farhi & Werning (2012 JPE): welfare gains from its implementation are small.
  - Difficult to solve with persistent idiosyncratic shocks. Farhi & Werning (2012 REstud) and Golosov, Troshkin & Tsyvinski (2014) in partial equilibrium settings find that restrictions to linear taxes lead to small welfare losses.
  - Even if only as a benchmark to more elaborate tax systems, it is useful to understand the properties of a simpler optimal tax system in a quantitative general equilibrium environment.