

The Retirement Gap: Entrepreneurs, Tax Policy, and the Cost of Liquidity

S. Dyrda¹, B. Robinson², R. Zimmermann¹ | ¹University of Toronto,²Western University

Sciences Po
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PRELIMINARY

The Issue: The Retirement Gap

Introduction

- Retirement accounts hold **33% of U.S. financial assets** (SCF 2019)
- Offer significant tax benefits: deferred growth, deductible contributions
- Yet **entrepreneurs allocate notably less** to retirement accounts compared to non-entrepreneurs

Why does this matter?

- Entrepreneurs: **12% of the population**, 30% of income, 40% of wealth
- Private businesses: generate **75% of U.S. value added**, closely tied to owners' financial choices

Key question:

What are the macroeconomic implications of this retirement gap?

Our Contribution: What We Do

Introduction

1. We exploit Survey of Consumer Finances (SCF) and Survey of Income and Program Participation (SIPP) to **document the retirement gap** in the data.
2. We develop a life-cycle, general equilibrium model featuring heterogeneous entrepreneurs and non-business owners (workers) that **rationalizes the retirement gap** through an explicit economic mechanism.
3. We **quantify the economic implications** of this asset allocation misalignment and derive its consequences for macroeconomic policy.

Our Contribution: What We Find

Introduction

1. **Entrepreneurs allocate**, on average, **8.0 ppt less** of their total assets to retirement accounts than non-business owners—robust across age and wealth groups.
2. Core trade-off: **tax perks vs. liquidity**
 - Liquidity finances capital when credit is tight.
 - Entrepreneurs face higher risk - liquidity helps insure consumption.
 - Retirement accounts are illiquid but tax-favoured.
3. Quantification:
 - The model reproduces about **half of the observed retirement-asset gap**.
 - Making retirement balances pledgeable can raise the aggregate capital stock by **up to 3 %** and allocative efficiency by **up to 0.6 %**.

Our Contribution: Where We Fit

Introduction

1. **Macroeconomics of Entrepreneurship:** Quadrini (2000), Cagetti and De Nardi (2006), Buera (2009), Buera and Kaboski (2012), Midrigan and Xu (2014), Moll (2014), Kochen (2022) etc.
 - Integrate financial frictions and retirement account decisions into a life-cycle model of entrepreneurship.
2. **Macro Public Finance:** Conesa and Krueger (1999), Conesa, Kitao, and Krueger (2009), McGrattan and Prescott (2017), Dyrda and Pugsley (2019), Brüggemann (2021), Boar and Midrigan (2023) etc.
 - Develop a unified framework to study jointly taxation of private businesses and the design of retirement accounts.
3. **Empirical Literature on Social Security:** Amromin and Smith (2003), Argento, Bryant, and Sabelhaus (2015), Goodman, Mortenson, Mackie, and Schramm (2021), and Stuart and Bryant (2024).
 - Document the retirement gap between workers and entrepreneurs in the U.S.

Outline

Introduction

1. Background and Empirics
2. Economic Environment
3. Taking Model to Data
4. Quantitative Experiments
5. Conclusion

Background and Empirics

Institutional Background: Tax Regimes for Saving and Investment

Empirics

	Contributions are Not Tax-deductible	Contributions are Tax-deductible
Tax on Returns	Ordinary Savings Accounts Stocks Bonds	Traditional IRAs and 401(k)s Defined Benefit Pensions
No Tax on Returns	Roth IRAs Owner-Occupied Homes Tax-exempt Bonds and 529 Plans	Health Savings Accounts DIY Home Improvements Employer-provided Health Insurance

Institutional Background: Traditional IRAs and 401(k)s

Key features:

- Both subject to **annual contribution limits**:
 - Under 50: \$7,000 (IRA), \$23,500 (401(k))
 - Catch-Up (Age 50+): +\$1,000 (IRA), +\$7,500 (401(k))
 - Employer contributions (401(k) only): combined limit \$70,000 (\$77,500 if 50+)
- Early withdrawal: **10% penalty + ordinary income tax** if age < 59½ (exceptions apply)
- Entrepreneurs vs. Workers:
 - Workers:
 - Often automatically enrolled in employer's 401(k) plan
 - Receive matching contributions, centralized plan administration
 - Entrepreneurs:
 - Must choose and administer their own vehicle (IRA, SEP-IRA, Solo 401(k))
 - If they have employees, they must cover staff under the same plan (adds complexity)

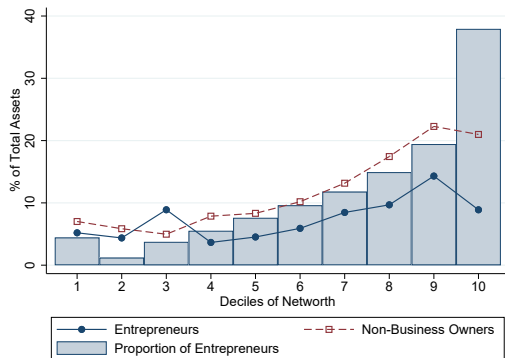
Data and Definitions

Empirics

- **Data:** 2010 - 2022 Survey of Consumer Finances (SCF)
- **Entrepreneurs:** households that own and actively manage a pass-through business
We exclude passive owners of businesses and owners of C-corporations
- **Non-business owners:** households that own no businesses
- **Net-worth:** the sum of all assets (businesses, housing, vehicles, other non-financial assets, and all financial assets) less all debts (mortgages, vehicle loans, student loans, credit cards, and other debts)
- **Retirement account balances:** the total balances of registered retirement accounts (IRA, Roth, 401k)

Retirement Account Balances over Wealth

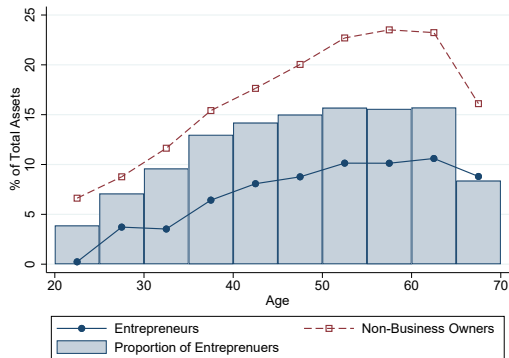
Empirics



- Share of assets invested in retirement accounts is lower for entrepreneurs
- Bigger difference among the wealthiest households (12 pts)
- The average gap is 4.4 pts across wealth groups

Retirement Account Balances over Age

Empirics



- Entrepreneurs at all ages invest less of their assets in their retirement accounts
- Bigger differences from 50-65 (13 pts)
- The average gap is 9.5 pts across age groups

Non-Business Assets

Entrepreneurs have Less Assets Invested in Retirement Accounts

Empirics

$$\frac{\text{Retirement Account Balance}_i}{\text{Total Assets}_i} = \beta_0 + \beta_1 \text{Entrepreneur}_i + \beta X + \epsilon$$

	(1) No Controls	(2) Wealth	(3) Age	(4) Both Wealth and Age
Entrepreneur	-0.00987* (0.00409)	-0.0673*** (0.00436)	-0.0219*** (0.00406)	-0.0799*** (0.00446)
Constant	Yes	Yes	Yes	Yes
Net Worth Deciles	No	Yes	No	Yes
Age Groups	No	No	Yes	Yes
Observations	27,479	27,479	27,479	27,479

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Entrepreneurs have 8.0 percentage points less of their assets invested in retirement accounts.

Entrepreneurs have Less Assets Invested in Retirement Accounts

Empirics

$$\frac{\text{Retirement Account Balance}_i}{\text{Total Assets}_i - \text{Business Value}_i} = \beta_0 + \beta_1 \text{Entrepreneur}_i + \beta X + \epsilon$$

	(1) No Controls	(2) Wealth	(3) Age	(4) Both Wealth and Age
Entrepreneur	0.0179*** (0.00450)	-0.0421*** (0.00469)	0.00568 (0.00451)	-0.0550*** (0.00480)
Constant	Yes	Yes	Yes	Yes
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Entrepreneurs have Less Assets Invested in Retirement Accounts

Empirics

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Age Groups	No	No	Yes	Yes
Observations	27,478	27,478	27,478	27,478

Entrepreneurs have 5.5 percentage points less of their non-business assets invested in retirement accounts.

Additional Empirical Evidence

- SIPP (panel dimension, preliminary)
 - Within-person fixed-effects suggest retirement-asset shares **dip** when the household head becomes an entrepreneur and rebound after returning to paid work.
 - **Gap opens slightly before** start-up and narrows after exit—consistent with forward-looking liquidity motives, but still suggestive.
- Finnish threshold reform (Benzarti, Harju, Matikka (2020))
 - Relaxing mandatory pension contributions (ownership rule 50%→30%) **cut contributions by ~16 %** and lifted young-firm sales by ~11 %.
 - **Strong age heterogeneity**: owners of mature firms channel freed cash into stock-market assets—no boost to sales or employment—implying the mandate mainly binds liquidity-constrained young ventures.

The Model

Demographics

Model

- Unit of analysis: household.
- Each period: a cohort of households is born at age $j = 1$.
- Finite lifespan with probabilistic death:
 - Households die at $j = J$ with certainty.
 - Face a death probability $1 - \phi_j$ at age j .
 - Cumulative survival probability to age j : $\Phi_j = \prod_{s=1}^j \phi_s$.
- Types:
 - At birth: fraction μ are entrepreneurs (e), $1 - \mu$ are workers (ω).
 - Both retire (r) exogenously: $J_R < J$.

Individual States

Model

- A household is a vector of individual states: $s = \{j, o, z_i, a, b\}$ where:

- Age: $j \in \{1, \dots, J\}$
- Occupation/Type: $i \in \{e, \omega, r\}$
- Productivity: z_i
- Liquid assets: $a \in [\underline{a}, \infty)$
- Retirement account balance $b \in [0, \infty)$

- Productivity

$$z_i = \varepsilon_{ij} \times m_i$$

where:

- m_i : a Markov process defined on $(M_i, \mathbb{B}(M_i))$ with transition matrix $Q_i : M_i \rightarrow M_i$.
- ε_{ij} : deterministic age-dependent efficiency component.

Individual Retirement Accounts (IRAs) and 401(k)s

Model

- Law of motion for retirement account balances:

$$b' = (1 + r^b)b + q \quad (\text{RetLoM})$$

→ r^b : return on the retirement account

→ $q \in [-b(1 + r^b), \theta_j y^{avg}]$: contribution/withdrawals at age j restricted by contribution limit θ_j

- Tax liability $T(y^{tax}, q)$ affected by q :

$$T(y^{tax}, q) = \underbrace{\mathcal{T}(y^{tax} - q)}_{\text{Contributions lower taxable income } y^{tax}} + \underbrace{\mathbf{1}_{q < 0, j < J_b} \tau_b q}_{\text{Penalty } \tau_b \text{ for early withdrawal prior to } J_b \text{ age}} \quad (\text{TaxLiab})$$

→ y^{tax} : taxable income

→ $\mathcal{T}(\cdot)$: individual income tax schedule

Preferences

Model

Lifetime expected utility

$$U(c, h) \equiv \mathbb{E} \left[\sum_{j=1}^J \beta^j \Phi_j u(c_j, h_j) \right]$$

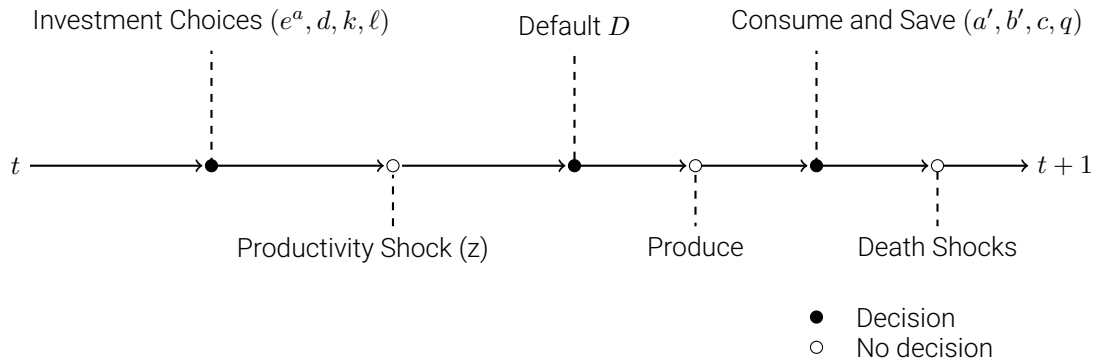
with

$$u(c_j, h_j) = \frac{\left(c_j^\gamma (1 - 1_{o \in \{w\}} h_j)^{1-\gamma} \right)^{1-\sigma}}{1 - \sigma}$$

- c_j : consumption
- h_j : hours worked
- γ : controls the consumption share relative to leisure;
- σ : governs risk aversion and the intertemporal elasticity of substitution;
- β : household's discount factor

Entrepreneurs: timing within a period

Model



Entrepreneurs: Debt Constraint, Value of the Firm and Default

Model

- Entrepreneur with state $s = (j, e, z_e, a, b)$ faces equity/debt constraint:

$$k = e^a + d \quad (\text{DebtCon})$$

→ k : capital

→ $e^a \in [0, a]$: equity in the business; d : debt at state contingent rate $r^d(s)$

- Value of the firm:

$$v(z'_e) = \overbrace{f(k, l, z'_e) - (1 + \tau_{EI})wl - \delta k - r^d(s)d}^{\pi(k, l, z'_e): \text{entrepreneurial profits}} + k - d$$

→ l : labor, τ_{EI} : employment insurance tax

- Default $D(z'_e) = 1$ scraps the value of the firm:

$$v^D(s, z'_e) = (1 - D(z'_e)) v(s, z'_e) \quad (\text{FirmVal})$$

→ **Limited liability**: entrepreneur is liable up to equity invested

Entrepreneurs: Dynamic Program

Model

$$V_j^e(z_e, a, b) = \max_{e^a \in [0, a], d, k, \ell} \sum_{z'_e} V_j^e(z'_e, a, b) Q(z'_e, z_e)$$

where

$$V_j^e(z'_e, a - e^a, b) = \max_{a' \geq \underline{a}, b', c, q, D} u(c) + \phi_j \beta [1_{j < J_R} V_{j+1}^e(z'_e, a', b') + 1_{j \geq J_R} V_{j+1}^r(y^{ret}, a', b')]$$

s.t. (RetLoM), (TaxLiab), (DebtCon), (FirmVal) and

$$x(z'_e) = v^D(z'_e) + (1 + r^a)(a - e^a)$$

$$c + a' + q = x(z'_e) - T(y^{tax}, q)$$

$$y^{tax} = \pi(k, l, z'_e) + r^a(a - e^a)$$

- $x(z'_e)$: cash-on-hand
- y^{tax} : taxable income. All entrepreneurs are pass-throughs.
- y^{ret} : retirement income

Workers and Retirees: Dynamic Program

Model

Workers:

$$\begin{aligned} V_j^w(z_w, a, b) &= \max_{a' \geq \underline{a}, b', c, h, q} u(c, h) + \phi_j \beta [1_{j < J_R} V_{j+1}^e(z'_e, a', b') + 1_{j \geq J_R} V_{j+1}^r(y^{ret}, a', b')] \\ \text{s.t.} & \quad (\text{RetLoM}), (\text{TaxLiab}) \text{ and} \\ c + a' + q &= whz_w + (1 + r^a)a \\ y^{tax} &= whz_w + r^a a. \end{aligned}$$

Retirees:

$$\begin{aligned} V_j^r(y^{ret}, a, b) &= \max_{a' \geq \underline{a}, b', c, q} u(c) + \phi_j \beta V_{j+1}^r(y^{ret}, a', b') \\ \text{s.t.} & \quad (\text{RetLoM}), (\text{TaxLiab}) \text{ and} \\ c + a' + q &= y^{ret} + (1 + r^a)a \\ y^{tax} &= y^{ret} + r^a a. \end{aligned}$$

Creditor's Problem and Interest Rate Determination

Model

- A competitive, risk-neutral creditor sets debt price $r_d(s)$ interest rate to break even:

$$1 + r^d(s) = \frac{(1 + r^a) - \left(\frac{\xi k(s)}{d(s)}\right) \sum_{z'_e} D(s, z'_e) Q(z'_e, z_e)}{\sum_{z'_e} (1 - D(s, z'_e)) Q(z'_e, z_e)}$$

- $D(z'_e) \in \{0, 1\}$: Default decision (default = 1)
 - $\xi \in [0, 1]$: Recovery rate upon default (legal liquidation)
 - k : Entrepreneur's total capital (collateral)
- Legal consistency:
 - Reflects partial creditor recovery typical in U.S. bankruptcy (Chapter 7).
 - Interest rates embed default risk and liquidation recovery explicitly.

Public Equity and Aggregation

Model

- Representative public C corporation solves the standard problem:

$$\max_{k_p, \ell_p} \Pi_p = A_p k_p^\alpha \ell_p^{1-\alpha} - w \ell_p - (r^b + \delta) k_p$$

- Aggregation:

- $\rightarrow \lambda_w : \mathcal{B}(\mathbf{S}_w) \rightarrow [0, 1]$: a measure of workers over state space $\mathbf{S}_w = (\mathbf{J} \times \mathbf{Z}_w \times \mathbf{A} \times \mathbf{B})$
- $\rightarrow \lambda_r : \mathcal{B}(\mathbf{S}_r) \rightarrow [0, 1]$: a measure of retirees over state space $\mathbf{S}_r = (\mathbf{J} \times \mathbf{A} \times \mathbf{B})$
- $\rightarrow \lambda_e : \mathcal{B}(\mathbf{S}_e) \rightarrow [0, 1]$: a measure of entrepreneurs over state space $\mathbf{S}_e = (\mathbf{J} \times \mathbf{Z}_e \times \mathbf{A} \times \mathbf{B})$
- $\rightarrow \lambda_e^{post} : \mathcal{B}(\mathbf{S}_e \times \mathbf{Z}_e) \rightarrow [0, 1]$: a measure of entrepreneurs post realization of z'_e , where

$$\lambda_e^{post}(j, z_e, a, b, z'_e) = \lambda_e(j, z_e, a, b) \times Q(z'_e, z_e)$$

- $\rightarrow \lambda : \mathcal{B}(\mathbf{S}) \rightarrow [0, 1]$: a measure of all households over $\mathbf{S} = (\mathbf{J} \times \mathbf{O} \times \mathbf{Z} \times \mathbf{A} \times \mathbf{B})$

Government's Budget

Model

- Budget constraint:

$$\int_S T(y^{tax}, q) d\lambda + A^d = \int_{S_r} y^{ret} d\lambda_r + G + N^{new} a_0$$

→ Assets of the deceased are:

$$A^d = \int_S (1 - \phi_j) (a'(s) + b'(s)) d\lambda(s)$$

→ Tax function follows HSV:

$$\mathcal{T}(y) = y - \lambda y^{1-\tau} + 1_{j < J_R} \tau_{ss} \min\{y, 2.47\bar{y}\}$$

- The employment insurance tax has to satisfy:

$$(1 + \tau_{EI}) = \frac{\int l d\lambda_e}{\int (1 - D(z'_e)) l d\lambda_e^{post}}$$

Market Clearings

Model

- Labor market clearing:

$$\int_{\mathbf{S}_w} h z_w d\lambda_w = \int_{\mathbf{S}_e} l d\lambda_e + l_p$$

- Safe asset market clearing:

$$\int_{\mathbf{S}_r} a d\lambda_r + \int_{\mathbf{S}_w} a d\lambda_w + \int_{\mathbf{S}_e} (a - e^a) d\lambda_e = \int_{\mathbf{S}_e} d d\lambda_e$$

- Public equity market (if clears):

$$\int_{\mathbf{S}} b d\lambda = k_p$$

otherwise add Π_p somewhere (e.g. government).

- Goods market clearing:

$$C + G + K' - (1 - \delta)K = Y + \int_{S_e \times Z_e} D\xi k d\lambda_e^{post}$$

Taking the Model to the Data

A Calibration Strategy

Taking the Model to the Data

Three sets of parameters:

1. Demographics, preferences and technology: $\left\{ \{\phi_j\}_{j=1}^J, \mu, \beta, \gamma, \sigma, \alpha, \eta, \zeta \right\}$
 - Demographic data
 - Macro moments
 - Average Hours Worked and aggregate Frisch elasticity
2. Productivity processes and life-cycle components: $\left\{ \{\varepsilon_{ij}\}_{j=1}^J, m_i \right\}$
 - Indirect inference to match life-cycle income profiles: Bhandari, Kass, McGrattan, and Schulz (2024).
 - Moments on income risk of workers and entrepreneurs: DeBacker et al. (2023)
3. Tax system and retirement accounts: $\left\{ \mathcal{T}(\cdot), \tau_b, \theta_j, J_b, y^{ret}, G \right\}$
 - Fiscal aggregates
 - Institutional details of retirement accounts

Demographics, preferences and technology

Taking the Model to the Data

- DRS technology for the entrepreneurs:

$$f(k, l, z'_e) = z_e^{1-\eta} (k^\alpha \ell^{1-\alpha})^\eta$$

- Parameters set exogenously:

- Model age: 25 to $J = 99$ with $J_R = 65$.
- $\{\phi_j\}_{j=1}^J$: matched to U.S. Social Security Administration (SSA) Life Tables.
- $\mu = 0.12$: fraction of entrepreneurs in the SCF data
- $\xi = 0.60$: 60% average recovery on senior secured loans to U.S. middle-market firms (S&P).
- $\alpha = 0.35$: capital share
- $\eta = 0.85$: standard value in macro entrepreneurship literature

- $\{\beta, \gamma, \sigma\}$ disciplined by the macro moments:

- wealth to output ratio: 2.6
- average hours worked: 0.26
- average Frisch elasticity: 1.0

Calibration: Income Risk

Taking the Model to the Data

- Workers - **mixture of normals**:

$$m'_\omega = \rho_\omega m_\omega + u_\omega.$$

The innovation term u_ω follows a two-component mixture of normal distributions:

$$u_\omega \sim \begin{cases} \mathcal{N}(0, \sigma_{\omega,1}^2) & \text{with probability } p_1, \\ \mathcal{N}(0, \sigma_{\omega,2}^2) & \text{with probability } p_2 = 1 - p_1. \end{cases}$$

- Entrepreneurs - **tweaked AR(1) process**:

$$m'_e = \rho_e m_e + u_e,$$

where u_e is drawn from *Tukey g-h* distribution that transforms a standard normal x to

$$x \exp \left((h/2)x^2 \right) (1 - 2h)^{3/4}.$$

→ thickness of the tails rises with h with standard normal having $h = 0$.

- Six parameters** to identify: $\{\rho_e, \rho_\omega, \sigma_e, \sigma_{\omega,1}, \sigma_{\omega,2}, h\}$.

Calibration: Income Risk

Taking the Model to the Data

Statistic	Data	Model
	%	
Standard deviation of 1-year earnings changes	22.5	23.8
Pct of workers with 1-year earnings changes in $[-10\%, 10\%]$	51.6	49.3
Pct of workers with 1-year earnings changes $> 50\%$	7.8	8.5
Standard deviation of 1-year business income changes	72.0	70.5
Pct of entrepreneurs with 1-year income changes in $[-10\%, 10\%]$	13.7	15.0
Pct of entrepreneurs with 1-year income changes $> 50\%$	29.2	30.8

Calibration: Income Risk

Taking the Model to the Data

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Entrepreneurs bear notably higher volatility in income fluctuations compared to workers.

Calibration: Income Risk

Taking the Model to the Data

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Entrepreneurs face frequent extreme income shocks, underscoring substantial risk exposure relative to workers.

Tax System and Retirement Accounts

Taking the Model to the Data

Tax schedule $\mathcal{T}(y_b)$

- $\tau = 0.15$ matches progressivity estimate in HSV.
- $\tau_{ss} = 12.4\%$ payroll rate in the US, earnings cap at $2.47\bar{y}$ (SSA wage base).
- $y^{\text{ret}} = 0.30\bar{y}$: matches the 30 % net replacement ratio implied by SSA data.
- λ set so federal $G/Y = 10\%$.

Retirement-account constraints

- Early-withdrawal penalty: $\tau_b = 10\%$ (U.S. rule before age $59\frac{1}{2}$).
- Contribution caps:

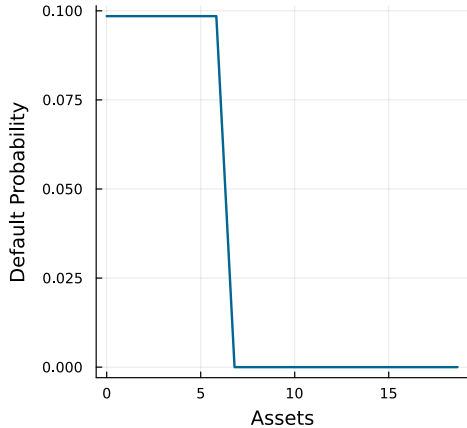
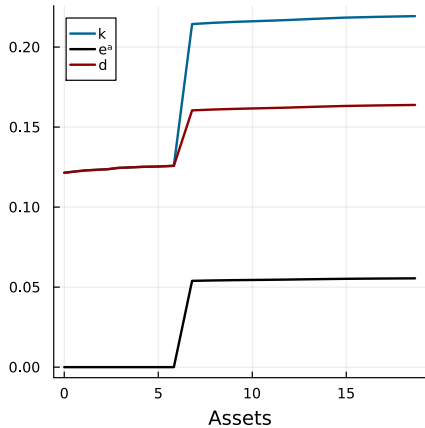
$$\theta_j = \begin{cases} 0.225 y_b, & y_b < \bar{y} \\ 23,500, & y_b \geq \bar{y} \end{cases} \quad \theta_j^e = \min(0.25 y_b, 70,000)$$

→ Workers: IRS 401(k) limit \$23.5k (2025) or 22.5

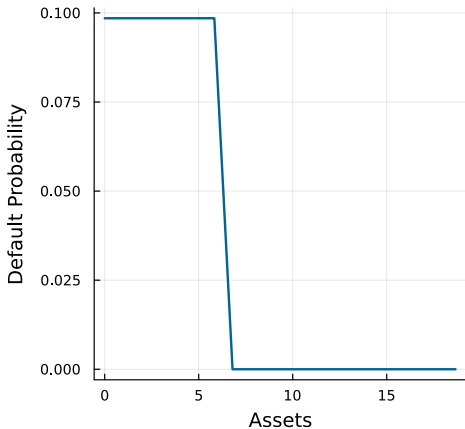
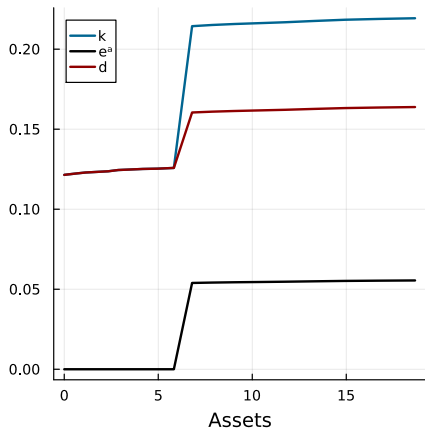
→ Entrepreneurs: SEP/Solo 401(k) limit = 25

Economic Mechanism and Model Validation

Capital, Debt, Equity and Default

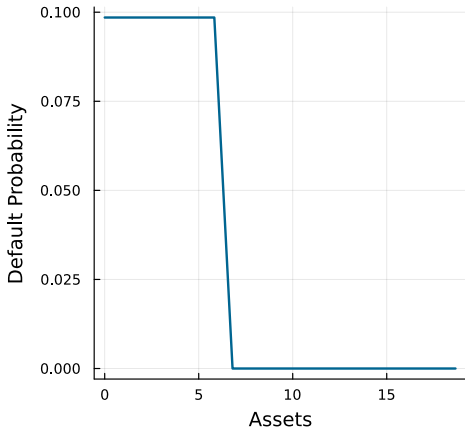
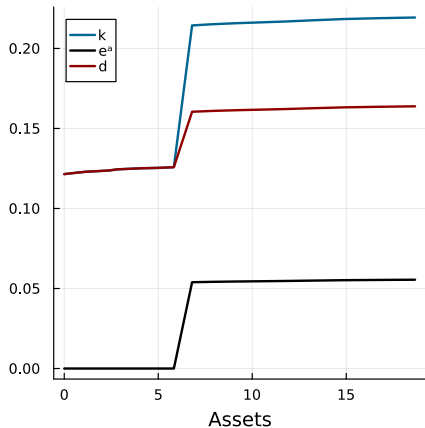


Capital, Debt, Equity and Default



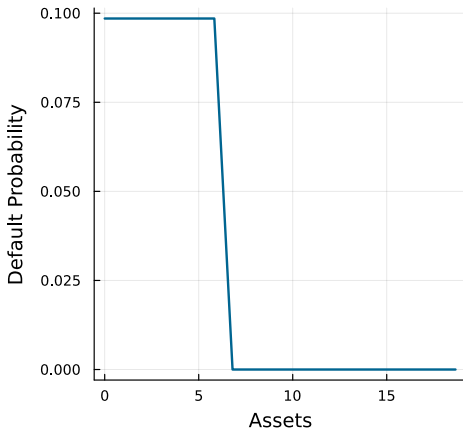
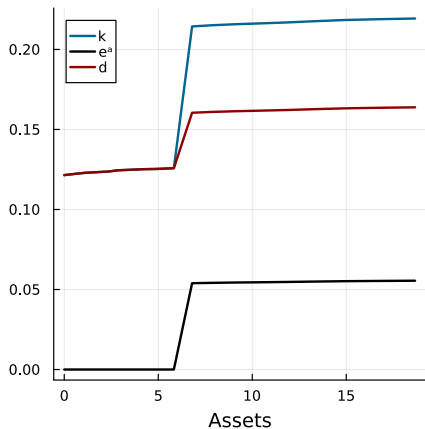
Two regimes: high default and no default. Default probability falls monotonically in-between.

Capital, Debt, Equity and Default



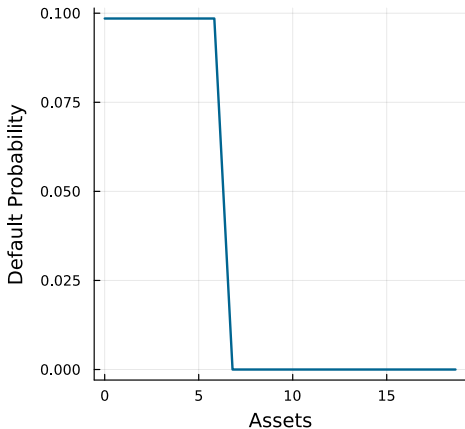
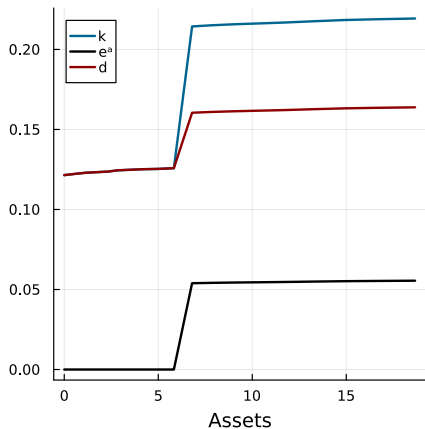
With very little wealth, putting in equity is futile—any equity would be wiped out in default, so they rely entirely on debt.

Capital, Debt, Equity and Default



As wealth increases, entrepreneurs begin to put in equity alongside debt, which cushions the firm and steadily lowers default probability.

Capital, Debt, Equity and Default



Once wealth exceeds a critical level, entrepreneurs fully eliminate default risk by investing enough equity and borrowing safely.

The Key Economic Tradeoff: tax benefits vs. liquidity needs

- Let $\tau = \{\tau_1, \tau_2, \tau_b\}$, and define marginal benefit of new capital:

$$G(y^{tax}, q; s, \tau) \equiv \sum_{z'_e} Q(z'_e, z_e) u_c(z'_e) (1 - D(z'_e)) \left[1 + (f_k - \delta) (1 - \mathcal{T}'(y^{tax}(\cdot) - q)) \right],$$

- Net marginal cost of debt:

$$C_d(y^{tax}, q; s, \tau) \equiv \sum_{z'_e} Q(z'_e, z_e) u_c(z'_e) (1 - D(z'_e)) \left[(1 + r^d(s)) (1 - \mathcal{T}'(y^{tax} - q)) \right].$$

- Net return to retirement contributions:

$$R^q(y^{tax}, q; s, \tau) = (1 + r^b) (1 + \mathcal{T}'(y^{tax} - q)).$$

- Entrepreneur's interior optimum equates:

$$R^e(y^{tax}, q; s, \tau) \equiv G(y^{tax}, q; s, \tau) - C_d(y^{tax}, q; s) = R^q(y^{tax}, q; s, \tau).$$

Comparative Statics: Early-Withdrawal Penalty τ_b

- Why τ_b matters: the key parameter that governs the illiquidity of retirement accounts.

Lemma

As τ_b increases:

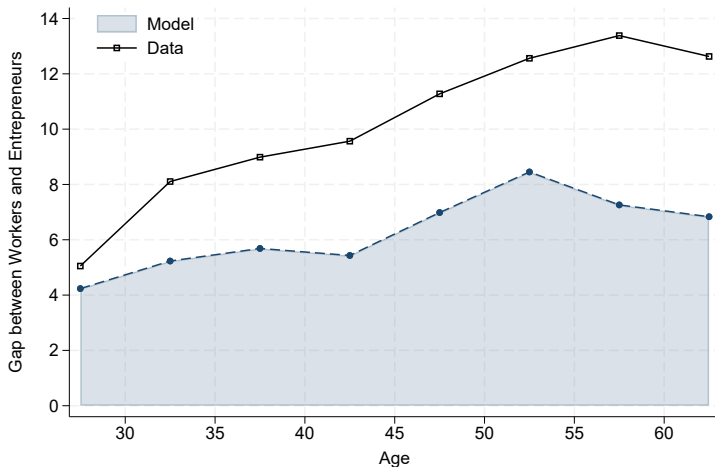
$$\frac{\partial y^{\text{tax}}}{\partial \tau_b} < 0, \quad \frac{\partial |q|}{\partial \tau_b} < 0.$$

That is, both gross taxable income and the magnitude of net withdrawals fall.

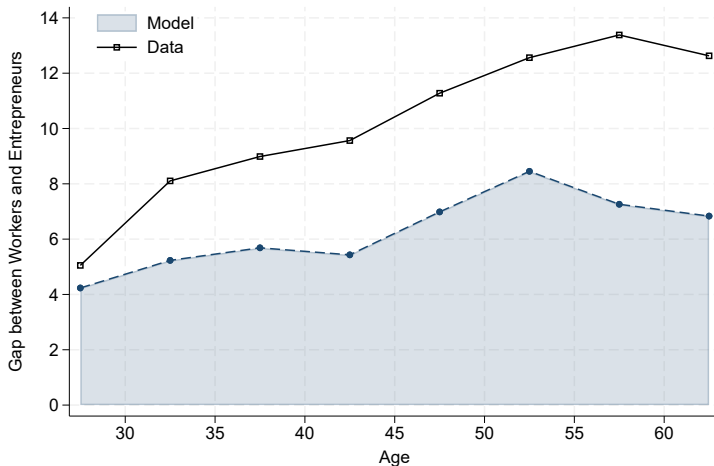
Mechanisms:

- Direct withdrawal effect: $\tau_b \uparrow \Rightarrow$ costlier withdrawals $\Rightarrow |q| \downarrow$.
- Liquidity channel: fewer withdrawals $\Rightarrow$ tighter financing $\Rightarrow k, n \downarrow \Rightarrow y^{\text{tax}} \downarrow$.
- Portfolio rebalancing: higher marginal value liquid assets, so even positive contributions $q > 0$ fall via forward-looking incentives.

Model Explains On Average Half of the Gap in Retirement Accounts



Model Explains On Average Half of the Gap in Retirement Accounts



Liquidity for production and self-insurance goes a long way toward explaining roughly half of entrepreneurs' retirement-saving gap.

Quantitative Experiments

Collateralizable Retirement Account (CRA)

- Under current IRS rules, retirement accounts cannot be pledged as collateral (IRC § 408(e)(4)).
- We introduce a policy parameter $\theta_b \in [0, 1]$: the fraction of b that can be pledged.
- The entrepreneur's equity/debt constraint still is:

$$k = e^a + d.$$

- Default now risks forfeiture of $\theta_b b$ and affects the law of motion:

$$b' = \left[(1 + r^b) - \underbrace{D(z'_e) (1 + r^b) \theta_b}_{\text{Seizure of the } \theta_b\text{-fraction of } b} \right] b + q.$$

Collateralizable Retirement Account (CRA)

- The interest rate price by the creditor's becomes then

$$1 + r^d(s) = \frac{(1 + r^a) - \left(\frac{\zeta k(s) + \min\{\theta_b b, d\}}{d(s)} \right) \sum_{z'_e} D(s, z'_e) Q(z'_e, z_e)}{\sum_{z'_e} (1 - D(s, z'_e)) Q(z'_e, z_e)}$$

- The $\min\{\theta_b b, d\}$ term ensures the lender cannot seize more collateral than the loan amount—avoiding “over-seizure” when $d < \theta_b b$.
- In default, the lender recovers $\zeta k + \min\{\theta_b b, d\}$, giving them a **senior claim** on up to $\theta_b b$.
- This relaxation of the borrowing constraint mechanically lowers the required r^d (or equivalently raises safe-debt capacity) for any given default probability.

Core Trade-off in CRA: Liquidity vs. Retirement Security

Liquidity Gains

- More collateral → looser credit.
- Firms scale up; capital and efficiency rise.
- Higher recovery trims loan spreads.

Retirement Risks

- Pension wealth exposed to business default.
- Extra leverage can push default up.
- Late-life shocks may drain pensions.

As θ_b rises:

- Capital, output, and leverage climb, peaking at a “sweet spot.”
- Spreads shrink early, then flatten as leverage grows.
- Default risk first falls, then rises; retirement-balance volatility keeps climbing.
- Portfolios tilt toward CRA, but households close to retirement cut exposure and keep more liquid cash.

Macro effects of CRA

Quantitative Exercise

	Collateralizable Retirement Accounts		
	$\theta_b = 0.25$	$\theta_b = 0.50$	$\theta_b = 0.75$
	% change vs. Baseline		
Output	0.6	1.1	0.8
Capital Stock	1.6	3.2	2.5
Allocative Efficiency	0.3	0.6	0.4
Leverage	2.1	5.3	7.4
Default (% of firms)	3.2	6.9	9.8
Loan spread (Δ bp)	-78	-158	-118
$Std(b; j > J_R)$	5.4	10.3	17.9
Ret. wealth adequacy at J_R	1.2	0.1	-3.9

Notes: " Δ bp" = basis-point change in the volume-weighted average loan spread (promised loan rate minus risk-free rate) relative to the baseline economy. Retirement-wealth adequacy is the mean ratio of retirement balance to lifetime earnings for survivors at J_R .

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Real side: Capital and output rise with pledgeability, peak around $\theta_b = 0.5$, then ease back—allocative efficiency follows the same hump-shape.

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Credit side: Leverage climbs steadily; loan spreads fall then flatten, and default shows the same U-shape—down first, up once leverage dominates.

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Household side: As more retirement balance is at risk, its volatility balloons and average adequacy slips below baseline.

Macro cost of illiquidity

Quantitative Exercise

	Withdrawal Penalty		
	$\tau_b = 0.00$	$\tau_b = 0.05$	$\tau_b = 0.20$
	% change vs. Baseline ($\tau_b = 0.10$)		
Output	0.6	0.3	-0.5
Capital Stock	1.7	0.9	-1.5
Allocative Efficiency	0.22	0.12	-0.28
Leverage (% change)	-2.4	-1.3	3.6
Default rate (level, %)	3.8	4.1	6.4
Loan spread (Δ bp)	-60	-35	85
$Std(b; j > J_R)$	11.0	5.3	9.2
Ret. wealth adequacy at J_R	2.1	0.8	-4.5

Notes: Output, capital, efficiency and leverage are The default row reports the level annual default rate. " Δ bp" = change in the volume-weighted average loan spread (promised rate – risk-free). $Std(b)$ and adequacy are computed for entrepreneur households only.

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Real side: Dropping the penalty unlocks internal funds—capital and output rise; steep penalties do the reverse.

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Credit side: Easier access cuts leverage, default, and spreads; tight penalties push them back up.

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Household side: Dispersion of b high with zero penalties (voluntary withdrawals) and with high penalties (seizures), while adequacy falls only in the high-illiquidity case.

Key Takeaways

What to Remember

1. **Retirement gap is real.**

Business owners consistently hold $\approx$ **8 pts less** in retirement accounts than other households.

2. **Why? Liquidity vs. tax shelter.**

Entrepreneurs need liquid wealth to fund capital and buffer risk, but tax-favoured accounts are locked up. That simple trade-off explains roughly **half** of the observed gap in our model.

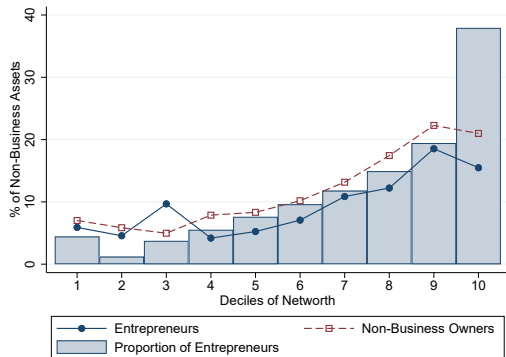
3. **Collateralising pensions can help—but only to a point.**

Letting retirees pledge **part** of their balance eases credit frictions and raises investment; pledge too much and rising leverage eats the gains.

Appendix

Retirement Account Balances over Wealth

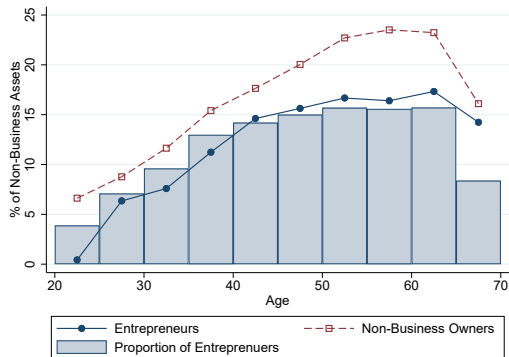
Empirics



- Share of non-business assets invested in retirement accounts is lower for entrepreneurs
- Gap is smaller when excluding the value of businesses
- Not clear that we should exclude the value of businesses: they are valuable assets

Retirement Account Balances over Age

Empirics



- Share of non-business assets invested in retirement accounts is lower for entrepreneurs
- Gap is smaller when excluding the value of businesses
- Not clear that we should exclude the value of businesses: they are valuable assets

Entrepreneurs have Less Assets Invested in Retirement Accounts

Empirics

$$\frac{\text{Retirement Account Balance}_i}{\text{Total Assets}_i - \text{Business Value}_i} = \beta_0 + \beta_1 \text{Entrepreneur}_i + \beta X + \epsilon$$

	(1) No Controls	(2) Wealth	(3) Age	(4) Both Wealth and Age
Entrepreneur	0.0179*** (0.00450)	-0.0421*** (0.00469)	0.00568 (0.00451)	-0.0550*** (0.00480)
Constant	Yes	Yes	Yes	Yes
Net Worth Deciles	No	Yes	No	Yes
Age Groups	No	No	Yes	Yes
Observations	27,478	27,478	27,478	27,478

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Entrepreneurs have 5.5 percentage points less of their non-business assets invested in retirement accounts.

Appendix: Prohibition on Pledging Retirement Accounts I

- **IRC § 408(e)(4)** explicitly states that if an IRA owner “pledges part of the IRA as collateral for a loan, the portion so pledged shall be treated as distributed” .
- Any “borrowing from” or “pledging as security” of IRA assets is a prohibited transaction that voids the IRA’s tax-favored status as of January 1 of that year .
- Parallel rules under ERISA and IRC § 4975 for qualified plans forbid “lending money or extending credit between a plan and a disqualified person,” which includes the plan participant .
- IRS FAQs reinforce: “If the owner of an IRA borrows from the IRA, the IRA is no longer an IRA ... If the owner of an IRA pledges part of the IRA as collateral, the part of the IRA that is pledged is treated as distributed” .
- Practical guidance warns custodians and participants: “Don’t pledge your IRA for any loans—IRS rules do not allow you to pledge any part of your IRA as security. If you do, that amount is treated as a distribution” .
- **401(k) plans** may permit internal participant loans but expressly prohibit using plan assets as external collateral: “You can’t use your 401(k) account as collateral for a loan. IRS regulations prohibit it” .

Institutional Background: SEP-IRA and Solo 401(k)

- **SEP-IRA**

- For any business (with or without employees)
- Employer-only contributions up to 25% of net self-employment income (2025 cap \$66 000)
- Must contribute same percentage for all eligible employees

- **Solo 401(k)**

- Only if you (± spouse) are sole employees
- Employee deferral: \$23 500 (<50), \$31 000 (50+)
- Employer profit-sharing up to 25%; combined cap \$66 000 (\$73 500 if 50+)
- Loans permitted (50 % of vested balance or \$50 000 max)

- **If You Have Other Employees**

- Cannot use Solo 401(k)
- Options: standard 401(k), SEP-IRA, or SIMPLE IRA covering all employees

Calibration: Life-Cycle Income Profiles via Indirect Inference

1. Generate panel of agents, compute $y_{i,j}$ for $j = 25, \dots, 65$, assign groups g and estimate:

$$y_{i,a} = \alpha_i + \sum_{j=25}^a \gamma_{j,g(i)} + \varepsilon_{i,a}, \quad \mathbb{E}[\varepsilon_{i,a}] = 0.$$

2. Estimate age-effects:

$$\hat{\gamma}_{a,g} = \frac{1}{N_g^a} \sum_{i \in g} \Delta y_{i,a} = \frac{1}{N_g^a} \sum_{i \in g} (y_{i,a} - y_{i,a-1}) = \frac{1}{N_g^a} \sum_{i \in g} (\gamma_{a,g(i)} + \Delta \varepsilon_{i,a}).$$

3. Parametrize Shocks with Chebyshev polynomials $T_k(x_j)$ where $x_j = 2 \frac{j-25}{40} - 1$.

$$\varepsilon_{\omega j} = \sum_{k=0}^K c_{\omega k} T_k(x_j), \quad \varepsilon_{ej} = \sum_{k=0}^K c_{ek} T_k(x_j),$$

4. Calibrate:

$$\min_{c_{\omega k}, c_{ek}} \sum_{j=25}^{65} \sum_g (\hat{\gamma}_{j,g} - \gamma_{j,g}^{\text{emp}})^2.$$